# End of Chapter Solutions <br> Essentials of Corporate Finance $6^{\text {th }}$ edition Ross, Westerfield, and Jordan 

## Updated 08-01-2007

## CHAPTER 1 INTRODUCTION TO CORPORATE FINANCE

## Answers to Concepts Review and Critical Thinking Questions

1. Capital budgeting (deciding on whether to expand a manufacturing plant), capital structure (deciding whether to issue new equity and use the proceeds to retire outstanding debt), and working capital management (modifying the firm's credit collection policy with its customers).
2. Disadvantages: unlimited liability, limited life, difficulty in transferring ownership, hard to raise capital funds. Some advantages: simpler, less regulation, the owners are also the managers, sometimes personal tax rates are better than corporate tax rates.
3. The primary disadvantage of the corporate form is the double taxation to shareholders of distributed earnings and dividends. Some advantages include: limited liability, ease of transferability, ability to raise capital, and unlimited life.
4. The treasurer's office and the controller's office are the two primary organizational groups that report directly to the chief financial officer. The controller's office handles cost and financial accounting, tax management, and management information systems. The treasurer's office is responsible for cash and credit management, capital budgeting, and financial planning. Therefore, the study of corporate finance is concentrated within the functions of the treasurer's office.
5. To maximize the current market value (share price) of the equity of the firm (whether it's publicly traded or not).
6. In the corporate form of ownership, the shareholders are the owners of the firm. The shareholders elect the directors of the corporation, who in turn appoint the firm's management. This separation of ownership from control in the corporate form of organization is what causes agency problems to exist. Management may act in its own or someone else's best interests, rather than those of the shareholders. If such events occur, they may contradict the goal of maximizing the share price of the equity of the firm.
7. A primary market transaction.
8. In auction markets like the NYSE, brokers and agents meet at a physical location (the exchange) to buy and sell their assets. Dealer markets like Nasdaq represent dealers operating in
dispersed locales who buy and sell assets themselves, usually communicating with other dealers electronically or literally over the counter.
9. Since such organizations frequently pursue social or political missions, many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., providing their goods and services to society at the lowest possible cost. Another approach might be to observe that even a not-for-profit business has equity. Thus, an appropriate goal would be to maximize the value of the equity.
10. An argument can be made either way. At one extreme, we could argue that in a market economy, all of these things are priced. This implies an optimal level of ethical and/or illegal behavior and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. The following is a classic (and highly relevant) thought question that illustrates this debate: "A firm has estimated that the cost of improving the safety of one of its products is $\$ 30$ million. However, the firm believes that improving the safety of the product will only save $\$ 20$ million in product liability claims. What should the firm do?"
11. The goal will be the same, but the best course of action toward that goal may require adjustments due different social, political, and economic climates.
12. The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed $\$ 35$, then they should fight the offer from the outside company. If management believes that this bidder or other unidentified bidders will actually pay more than $\$ 35$ per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.
13. We would expect agency problems to be less severe in other countries, primarily due to the relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be able to implement more effective monitoring mechanisms than can individual owners, given an institutions' deeper resources and experiences with their own management. The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control.

## SOLUTIONS B-4

14. How much is too much? Who is worth more, Steve Jobs or Tiger Woods? The simplest answer is that there is a market for executives just as there is for all types of labor. Executive compensation is the price that clears the market. The same is true for athletes and performers. Having said that, one aspect of executive compensation deserves comment. A primary reason executive compensation has grown so dramatically is that companies have increasingly moved to stock-based compensation. Such movement is obviously consistent with the attempt to better align stockholder and management interests. In recent years, stock prices have soared, so management has cleaned up. It is sometimes argued that much of this reward is simply due to rising stock prices in general, not managerial performance. Perhaps in the future, executive compensation will be designed to reward only differential performance, i.e., stock price increases in excess of general market increases.
15. The biggest reason that a company would "go dark" is because of the increased audit costs associated with Sarbanes-Oxley compliance. A company should always do a cost-benefit analysis, and it may be the case that the costs of complying with Sarbox outweigh the benefits. Of course, the company could always be trying to hide financial issues of the company! This is also one of the costs of going dark: Investors surely believe that some companies are going dark to avoid the increased scrutiny from SarbOx. This taints other companies that go dark just to avoid compliance costs. This is similar to the lemon problem with used automobiles: Buyers tend to underpay because they know a certain percentage of used cars are lemons. So, investors will tend to pay less for the company stock than they otherwise would. It is important to note that even if the company delists, its stock is still likely traded, but on the over-the-counter market pink sheets rather than on an organized exchange. This adds another cost since the stock is likely to be less liquid now. All else the same, investors pay less for an asset with less liquidity. Overall, the cost to the company is likely a reduced market value. Whether delisting is good or bad for investors depends on the individual circumstances of the company. It is also important to remember that there are already many small companies that file only limited financial information already.

## CHAPTER 2 <br> WORKING WITH FINANCIAL STATEMENTS

## Answers to Concepts Review and Critical Thinking Questions

1. Liquidity measures how quickly and easily an asset can be converted to cash without significant loss in value. It's desirable for firms to have high liquidity so that they can more safely meet short-term creditor demands. However, liquidity also has an opportunity cost. Firms generally reap higher returns by investing in illiquid, productive assets. It's up to the firm's financial management staff to find a reasonable compromise between these opposing needs.
2. The recognition and matching principles in financial accounting call for revenues, and the costs associated with producing those revenues, to be "booked" when the revenue process is essentially complete, not necessarily when the cash is collected or bills are paid. Note that this way is not necessarily correct; it's the way accountants have chosen to do it.
3. Historical costs can be objectively and precisely measured, whereas market values can be difficult to estimate, and different analysts would come up with different numbers. Thus, there is a tradeoff between relevance (market values) and objectivity (book values).
4. Depreciation is a non-cash deduction that reflects adjustments made in asset book values in accordance with the matching principle in financial accounting. Interest expense is a cash outlay, but it's a financing cost, not an operating cost.
5. Market values can never be negative. Imagine a share of stock selling for $-\$ 20$. This would mean that if you placed an order for 100 shares, you would get the stock along with a check for $\$ 2,000$. How many shares do you want to buy? More generally, because of corporate and individual bankruptcy laws, net worth for a person or a corporation cannot be negative, implying that liabilities cannot exceed assets in market value.
6. For a successful company that is rapidly expanding, capital outlays would typically be large, possibly leading to negative cash flow from assets. In general, what matters is whether the money is spent wisely, not whether cash flow from assets is positive or negative.
7. It's probably not a good sign for an established company, but it would be fairly ordinary for a startup, so it depends.
8. For example, if a company were to become more efficient in inventory management, the amount of inventory needed would decline. The same might be true if it becomes better at collecting its receivables. In general, anything that leads to a decline in ending NWC relative to beginning NWC would have this effect. Negative net capital spending would mean more long-lived assets were liquidated than purchased.

## SOLUTIONS B-6

9. If a company raises more money from selling stock than it pays in dividends in a particular period, its cash flow to stockholders will be negative. If a company borrows more than it pays in interest, its cash flow to creditors will be negative.
10. The adjustments discussed were purely accounting changes; they had no cash flow or market value consequences unless the new accounting information caused stockholders to revalue the company.
11. The legal system thought it was fraud. Mr. Sullivan disregarded GAAP procedures, which is fraudulent. That fraudulent activity is unethical goes without saying.
12. By reclassifying costs as assets, it lowered costs when the lines were leased. This increased the net income for the company. It probably increased most future net income amounts, although not as much as you might think. Since the telephone lines were fixed assets, they would have been depreciated in the future. This depreciation would reduce the effect of expensing the telephone lines. The cash flows of the firm would basically be unaffected no matter what the accounting treatment of the telephone lines.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. The balance sheet for the company will look like this:

|  | Balance sheet |  |  |
| :--- | ---: | :--- | ---: |
| Current assets | $\$ 1,850$ | Current liabilities | $\$ 1,600$ |
| Net fixed assets | 8,600 | Long-term debt | 6,100 |
|  |  | Owner's equity | $\underline{2,750}$ |
|  |  |  |  |
| Total assets | $\underline{\$ 10,450}$ | Total liabilities \& Equity | $\underline{\$ 10,450}$ |

The owner's equity is a plug variable. We know that total assets must equal total liabilities \& owner's equity. Total liabilities and equity is the sum of all debt and equity, so if we subtract debt from total liabilities and owner's equity, the remainder must be the equity balance, so:

Owner's equity $=$ Total liabilities $\&$ equity - Current liabilities - Long-term debt
Owner's equity $=\$ 10,450-1,600-6,100$
Owner's equity $=\$ 2,750$
Net working capital is current assets minus current liabilities, so:
NWC $=$ Current assets - Current liabilities
NWC = \$1,850 - 1,600
NWC $=\$ 250$
2. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

|  | Income Statement |  |
| :--- | ---: | ---: |
| Sales | $\$ 625,000$ |  |
| Costs | 260,000 |  |
| Depreciation | 79,000 <br> EBIT | 436,000 <br> Interest |
| Taxable income | $\$ 243,000$ |  |
| Taxes | $\underline{85,050}$ |  |
| Net income | $\underline{\$ 157,950}$ |  |

3. The dividends paid plus addition to retained earnings must equal net income, so:

Net income $=$ Dividends + Addition to retained earnings
Addition to retained earnings $=\$ 157,950-60,000$
Addition to retained earnings $=\$ 97,950$
4. Earnings per share is the net income divided by the shares outstanding, so:

EPS $=$ Net income $/$ Shares outstanding
EPS $=\$ 157,950 / 40,000$
EPS $=\$ 3.95$ per share
And dividends per share are the total dividends paid divided by the shares outstanding, so:
DPS $=$ Dividends $/$ Shares outstanding
DPS $=\$ 60,000 / 40,000$
DPS $=\$ 1.50$ per share
5. To find the book value of assets, we first need to find the book value of current assets. We are given the NWC. NWC is the difference between current assets and current liabilities, so we can use this relationship to find the book value of current assets. Doing so, we find:

NWC $=$ Current assets - Current liabilities
Current assets $=\$ 100,000+780,000=\$ 880,000$

## SOLUTIONS B-8

Now we can construct the book value of assets. Doing so, we get:

## Book value of assets

| Current assets | $\$ 8880,000$ |
| :--- | ---: |
| Fixed assets | $\underline{4,800,000}$ |
| Total assets | $\underline{\$ 5,680,000}$ |

All of the information necessary to calculate the market value of assets is given, so:

## Market value of assets

| Current assets | $\$ 805,000$ |
| :--- | ---: |
| Fixed assets | $\underline{5,600,000}$ |
| Total assets | $\underline{\$ 6,405,000}$ |

6. Using Table 2.3, we can see the marginal tax schedule. The first $\$ 25,000$ of income is taxed at 15 percent, the next $\$ 50,000$ is taxed at 25 percent, the next $\$ 25,000$ is taxed at 34 percent, and the next $\$ 215,000$ is taxed at 39 percent. So, the total taxes for the company will be:

Taxes $=0.15(\$ 50,000)+0.25(\$ 25,000)+0.34(\$ 25,000)+0.39(\$ 315,000-100,000)$
Taxes $=\$ 106,100$
7. The average tax rate is the total taxes paid divided by net income, so:

Average tax rate $=$ Total tax $/$ Net income
Average tax rate $=\$ 106,100 / \$ 315,000$
Average tax rate $=.3368$ or $33.68 \%$
The marginal tax rate is the tax rate on the next dollar of income. The company has net income of $\$ 315,000$ and the 39 percent tax bracket is applicable to a net income of $\$ 335,000$, so the marginal tax rate is 39 percent.
8. To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

| Income Statement |  |
| :---: | :---: |
| Sales | \$16,550 |
| Costs | 5,930 |
| Depreciation | 1,940 |
| EBIT | \$8,680 |
| Interest | 1,460 |
| Taxable income | \$7,220 |
| Taxes (35\%) | 2,527 |
| Net income | \$4,693 |

Now we can calculate the OCF, which is:
OCF $=$ EBIT + Depreciation - Taxes
$\mathrm{OCF}=\$ 8,680+1,940-2,527$
OCF $=\$ 8,093$
9. Net capital spending is the increase in fixed assets, plus depreciation. Using this relationship, we find:

Net capital spending $=\mathrm{NFA}_{\text {end }}-\mathrm{NFA}_{\text {beg }}+$ Depreciation
Net capital spending $=\$ 2,120,000-1,875,000+220,000$
Net capital spending $=\$ 465,000$
10. The change in net working capital is the end of period net working capital minus the beginning of period net working capital, so:

Change in NWC $=\mathrm{NWC}_{\text {end }}-\mathrm{NWC}_{\text {beg }}$
Change in NWC $=\left(\mathrm{CA}_{\text {end }}-\mathrm{CL}_{\text {end }}\right)-\left(\mathrm{CA}_{\text {beg }}-\mathrm{CL}_{\text {beg }}\right)$
Change in NWC $=(\$ 910-335)-(840-320)$
Change in NWC $=\$ 55$
11. The cash flow to creditors is the interest paid, minus any new borrowing, so:

Cash flow to creditors $=$ Interest paid - Net new borrowing
Cash flow to creditors $=$ Interest paid $-\left(\mathrm{LTD}_{\text {end }}-\mathrm{LTD}_{\text {beg }}\right)$
Cash flow to creditors $=\$ 49,000-(\$ 1,800,000-1,650,000)$
Cash flow to creditors $=-\$ 101,000$
12. The cash flow to stockholders is the dividends paid minus any new equity raised. So, the cash flow to stockholders is: (Note that APIS is the additional paid-in surplus.)

Cash flow to stockholders $=$ Dividends paid - Net new equity
Cash flow to stockholders $=$ Dividends paid $-\left(\right.$ Common $_{\text {end }}+$ APIS $\left._{\text {end }}\right)-\left(\right.$ Common $_{\text {beg }}+$ APIS $\left._{\text {beg }}\right)$
Cash flow to stockholders $=\$ 70,000-[(\$ 160,000+3,200,000)-(\$ 150,000+2,900,000)]$
Cash flow to stockholders $=-\$ 240,000$
13. We know that cash flow from assets is equal to cash flow to creditors plus cash flow to stockholders. So, cash flow from assets is:

Cash flow from assets $=$ Cash flow to creditors + Cash flow to stockholders
Cash flow from assets $=-\$ 101,000-240,000$
Cash flow from assets $=-\$ 341,000$

## SOLUTIONS B-10

We also know that cash flow from assets is equal to the operating cash flow minus the change in net working capital and the net capital spending. We can use this relationship to find the operating cash flow. Doing so, we find:

Cash flow from assets $=$ OCF - Change in NWC - Net capital spending
$-\$ 341,000=$ OCF $-(-\$ 135,000)-(760,000)$
OCF $=-\$ 341,000-135,000+760,000$
$\mathrm{OCF}=\$ 284,000$

## Intermediate

14. $a$. To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

| Income Statement |  |
| :--- | ---: |
| Sales | $\$ 138,000$ |
| Costs | 71,500 |
| Other Expenses | 4,100 |
| Depreciation | 10,100 |
| EBIT | $\$ 52,300$ |
| Interest | 7,900 |
| Taxable income | $\$ 44,400$ |
| Taxes | $\underline{17,760}$ |
| Net income | $\underline{\$ 26,640}$ |


| Dividends | $\$ 5,400$ |
| :--- | :--- |
| Addition to retained earnings | 21,240 |

Dividends paid plus addition to retained earnings must equal net income, so:
Net income $=$ Dividends + Addition to retained earnings
Addition to retained earnings $=\$ 26,640-5,400$
Addition to retained earnings $=\$ 21,240$
So, the operating cash flow is:
OCF = EBIT + Depreciation - Taxes
$\mathrm{OCF}=\$ 52,300+10,100-17,760$
$\mathrm{OCF}=\$ 44,640$
$b$. The cash flow to creditors is the interest paid, minus any new borrowing. Since the company redeemed long-term debt, the new borrowing is negative. So, the cash flow to creditors is:

Cash flow to creditors $=$ Interest paid - Net new borrowing
Cash flow to creditors $=\$ 7,900-(-\$ 3,800)$
Cash flow to creditors $=\$ 11,700$
c. The cash flow to stockholders is the dividends paid minus any new equity. So, the cash flow to stockholders is:

Cash flow to stockholders $=$ Dividends paid - Net new equity
Cash flow to stockholders $=\$ 5,400-2,500$
Cash flow to stockholders $=\$ 2,900$
d. In this case, to find the addition to NWC, we need to find the cash flow from assets. We can then use the cash flow from assets equation to find the change in NWC. We know that cash flow from assets is equal to cash flow to creditors plus cash flow to stockholders. So, cash flow from assets is:

Cash flow from assets $=$ Cash flow to creditors + Cash flow to stockholders
Cash flow from assets $=\$ 11,700+2,900$
Cash flow from assets $=\$ 14,600$
Net capital spending is equal to depreciation plus the increase in fixed assets, so:
Net capital spending $=$ Depreciation + Increase in fixed assets
Net capital spending $=\$ 10,100+17,400$
Net capital spending $=\$ 27,500$
Now we can use the cash flow from assets equation to find the change in NWC. Doing so, we find:

Cash flow from assets $=$ OCF - Change in NWC - Net capital spending
$\$ 14,600=\$ 44,640-$ Change in NWC - $\$ 27,500$
Change in NWC $=\$ 2,540$
15. Here we need to work the income statement backward. Starting with net income, we know that net income is:

Net income $=$ Dividends + Addition to retained earnings
Net income $=\$ 915+2,100$
Net income $=\$ 3,015$
Net income is also the taxable income, minus the taxable income times the tax rate, or:
Net income $=$ Taxable income $-($ Taxable income $)($ Tax rate $)$
Net income $=$ Taxable income $(1-$ Tax rate $)$
We can rearrange this equation and solve for the taxable income as:
Taxable income $=$ Net income $/(1-$ Tax rate $)$
Taxable income $=\$ 3,015 /(1-.40)$
Taxable income $=\$ 5,025$

## SOLUTIONS B-12

EBIT minus interest equals taxable income, so rearranging this relationship, we find:

```
EBIT \(=\) Taxable income + Interest
EBIT \(=\$ 5,025+1,360\)
EBIT \(=\$ 6,385\)
```

Now that we have the EBIT, we know that sales minus costs minus depreciation equals EBIT. Solving this equation for EBIT, we find:

EBIT $=$ Sales - Costs - Depreciation
$\$ 6,385=\$ 42,000-28,000-$ Depreciation
Depreciation $=\$ 7,615$
16. We can fill in the balance sheet with the numbers we are given. The balance sheet will be:

Balance Sheet

| Cash | $\$ 167,000$ |  | Accounts payable | $\$ 236,000$ |
| :--- | ---: | :--- | ---: | ---: |
| Accounts receivable | 241,000 |  | Notes payable | $\underline{176,000}$ |
| Inventory | $\underline{498,000}$ | Current liabilities | $\$ 412,000$ |  |
| Current assets | $\$ 906,000$ | Long-term debt <br> Total liabilities | $\$ 1,325,000$ |  |
| Tangible net fixed assets | $\$ 4,700,000$ |  |  |  |
| Intangible net fixed assets | $\underline{818,000}$ |  | Common stock | $? ?$ |
| Total assets | $\underline{\$ 6,424,000}$ | Accumulated retained earnings <br> Total liabilities \& owners' equity | $\underline{\underline{\$ 6,230,424,000}}$ |  |

Owners' equity has to be total liabilities \& equity minus accumulated retained earnings and total liabilities, so:

Owner's equity $=$ Total liabilities \& equity - Accumulated retained earnings - Total liabilities
Owners' equity $=\$ 6,424,000-4,230,000-1,325,000$
Owners' equity $=\$ 869,000$
17. Owner's equity is the maximum of total assets minus total liabilities, or zero. Although the book value of owners' equity can be negative, the market value of owners' equity cannot be negative, so:

Owners' equity $=\operatorname{Max}[(\mathrm{TA}-\mathrm{TL}), 0]$
a. If total assets are $\$ 8,700$, the owners' equity is:

Owners' equity $=\operatorname{Max}[(\$ 8,700-7,500), 0]$
Owners' equity $=\$ 1,200$
b. If total assets are $\$ 6,900$, the owners' equity is:

Owners' equity $=\operatorname{Max}[(\$ 6,900-7,500), 0]$
Owners' equity $=\$ 0$
18. a. Using Table 2.3 , we can see the marginal tax schedule. For Corporation Growth, the first $\$ 50,000$ of income is taxed at 15 percent, the next $\$ 25,000$ is taxed at 25 percent, and the next $\$ 25,000$ is taxed at 34 percent. So, the total taxes for the company will be:

Taxes $_{\text {Growth }}=0.15(\$ 50,000)+0.25(\$ 25,000)+0.34(\$ 8,000)$
Taxes $_{\text {Growth }}=\$ 16,470$
For Corporation Income, the first $\$ 50,000$ of income is taxed at 15 percent, the next $\$ 25,000$ is taxed at 25 percent, the next $\$ 25,000$ is taxed at 34 percent, the next $\$ 235,000$ is taxed at 39 percent, and the next $\$ 7,965,000$ is taxed at 34 percent. So, the total taxes for the company will be:

Taxes $_{\text {Income }}=0.15(\$ 50,000)+0.25(\$ 25,000)+0.34(\$ 25,000)+0.39(\$ 235,000)$

$$
+0.34(\$ 7,965,000)
$$

$\operatorname{Taxes}_{\text {Income }}=\$ 2,822,000$
$b$. The marginal tax rate is the tax rate on the next $\$ 1$ of earnings. Each firm has a marginal tax rate of $34 \%$ on the next $\$ 10,000$ of taxable income, despite their different average tax rates, so both firms will pay an additional $\$ 3,400$ in taxes.
19. $a$. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

| Sales | $\$ 2,700,000$ |
| :--- | ---: |
| Cost of goods sold | $1,690,000$ |
| Other expenses | 465,000 |
| Depreciation | $\underline{530,000}$ |
| EBIT | $\$ 15,000$ |
| Interest | $\underline{210,000}$ |
| Taxable income | $-\$ 195,000$ |
| Taxes (35\%) | $\underline{0}$ |
| Net income | $\underline{\underline{195,000}}$ |

The taxes are zero since we are ignoring any carryback or carryforward provisions.
b. The operating cash flow for the year was:

```
OCF = EBIT + Depreciation - Taxes
OCF = $15,000 + 530,000-0
OCF =$545,000
```

c. Net income was negative because of the tax deductibility of depreciation and interest expense. However, the actual cash flow from operations was positive because depreciation is a non-cash expense and interest is a financing, not an operating, expense.

## SOLUTIONS B-14

20. A firm can still pay out dividends if net income is negative; it just has to be sure there is sufficient cash flow to make the dividend payments. The assumptions made in the question are:

Change in $\mathrm{NWC}=$ Net capital spending $=$ Net new equity $=0$

To find the new long-term debt, we first need to find the cash flow from assets. The cash flow from assets is:

Cash flow from assets $=\mathrm{OCF}-$ Change in NWC - Net capital spending
Cash flow from assets $=\$ 545,000-0-0$
Cash flow from assets $=\$ 545,000$

We can also find the cash flow to stockholders, which is:

Cash flow to stockholders $=$ Dividends - Net new equity
Cash flow to stockholders $=\$ 500,000-0$
Cash flow to stockholders $=\$ 500,000$

Now we can use the cash flow from assets equation to find the cash flow to creditors. Doing so, we get:

Cash flow from assets $=$ Cash flow to creditors + Cash flow to stockholders
$\$ 545,000=$ Cash flow to creditors $+\$ 500,000$
Cash flow to creditors $=\$ 45,000$
Now we can use the cash flow to creditors equation to find:

Cash flow to creditors $=$ Interest - Net new long-term debt
$\$ 45,000=\$ 210,000-$ Net new long-term debt
Net new long-term debt $=\$ 165,000$
21. $a$. To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

| Sales | $\$ 18,450$ |
| :--- | ---: |
| Cost of goods sold | 13,610 |
| Depreciation | $\underline{2,420}$ |
| EBIT | $\$ 2,420$ |
| Interest | $\underline{260}$ |
| Taxable income | $\$ 2,160$ |
| Taxes (35\%) | $\underline{\$ 1,404}$ |
| Net income |  |

b. The operating cash flow for the year was:

OCF = EBIT + Depreciation - Taxes
$\mathrm{OCF}=\$ 2,420+2,420-756=\$ 4,084$
c. To calculate the cash flow from assets, we also need the change in net working capital and net capital spending. The change in net working capital was:

Change in NWC $=\mathrm{NWC}_{\text {end }}-\mathrm{NWC}_{\text {beg }}$
Change in NWC $=\left(\mathrm{CA}_{\text {end }}-\mathrm{CL}_{\text {end }}\right)-\left(\mathrm{CA}_{\text {beg }}-\mathrm{CL}_{\text {beg }}\right)$
Change in NWC $=(\$ 4,690-2,720)-(\$ 3,020-2,260)$
Change in NWC $=\$ 1,210$
And the net capital spending was:
Net capital spending $=\mathrm{NFA}_{\text {end }}-\mathrm{NFA}_{\text {beg }}+$ Depreciation
Net capital spending $=\$ 12,700-12,100+2,420$
Net capital spending $=\$ 3,020$
So, the cash flow from assets was:
Cash flow from assets $=$ OCF - Change in NWC - Net capital spending
Cash flow from assets $=\$ 4,084-1,210-3,020$
Cash flow from assets $=-\$ 146$
The cash flow from assets can be positive or negative, since it represents whether the firm raised funds or distributed funds on a net basis. In this problem, even though net income and OCF are positive, the firm invested heavily in both fixed assets and net working capital; it had to raise a net $\$ 146$ in funds from its stockholders and creditors to make these investments.
d. The cash flow from creditors was:

Cash flow to creditors $=$ Interest - Net new LTD
Cash flow to creditors $=\$ 260-0$
Cash flow to creditors $=\$ 260$
Rearranging the cash flow from assets equation, we can calculate the cash flow to stockholders as:

Cash flow from assets $=$ Cash flow to stockholders + Cash flow to creditors
-\$146 = Cash flow to stockholders + \$260
Cash flow to stockholders $=-\$ 406$
Now we can use the cash flow to stockholders equation to find the net new equity as:
Cash flow to stockholders = Dividends - Net new equity
$-\$ 406=\$ 450-$ Net new equity
Net new equity $=\$ 856$

## SOLUTIONS B-16

The firm had positive earnings in an accounting sense ( $\mathrm{NI}>0$ ) and had positive cash flow from operations. The firm invested $\$ 1,210$ in new net working capital and $\$ 3,020$ in new fixed assets. The firm had to raise $\$ 146$ from its stakeholders to support this new investment. It accomplished this by raising $\$ 856$ in the form of new equity. After paying out $\$ 450$ in the form of dividends to shareholders and $\$ 260$ in the form of interest to creditors, $\$ 146$ was left to just meet the firm's cash flow needs for investment.
22. a. To calculate owners' equity, we first need total liabilities and owners' equity. From the balance sheet relationship we know that this is equal to total assets. We are given the necessary information to calculate total assets. Total assets are current assets plus fixed assets, so:

Total assets $=$ Current assets + Fixed assets $=$ Total liabilities and owners' equity
For 2007, we get:
Total assets $=\$ 2,050+9,504$
Total assets $=\$ 11,554$
Now, we can solve for owners' equity as:
Total liabilities and owners' equity $=$ Current liabilities + Long-term debt + Owners' equity $\$ 11,554=\$ 885+5,184+$ Owners' equity
Owners' equity $=\$ 5,485$
For 2008, we get:
Total assets $=\$ 2,172+9,936$
Total assets $=\$ 12,108$
Now we can solve for owners' equity as:
Total liabilities and owners' equity $=$ Current liabilities + Long-term debt + Owners' equity $\$ 12,108=\$ 1,301+6,048+$ Owners' equity
Owners' equity $=\$ 4,759$
$b$. The change in net working capital was:
Change in NWC $=\mathrm{NWC}_{\text {end }}-\mathrm{NWC}_{\text {beg }}$
Change in NWC $=\left(\mathrm{CA}_{\text {end }}-\mathrm{CL}_{\text {end }}\right)-\left(\mathrm{CA}_{\text {beg }}-\mathrm{CL}_{\text {beg }}\right)$
Change in NWC $=(\$ 2,172-1,301)-(\$ 2,050-885)$
Change in NWC $=-\$ 294$
c. To find the amount of fixed assets the company sold, we need to find the net capital spending, The net capital spending was:

Net capital spending $=\mathrm{NFA}_{\text {end }}-\mathrm{NFA}_{\text {beg }}+$ Depreciation
Net capital spending $=\$ 9,936-9,504+2,590$
Net capital spending $=\$ 3,022$

To find the fixed assets sold, we can also calculate net capital spending as:
Net capital spending $=$ Fixed assets bought - Fixed assets sold
$\$ 3,022=\$ 4,320$ - Fixed assets sold
Fixed assets sold $=\$ 1,298$
To calculate the cash flow from assets, we first need to calculate the operating cash flow. For the operating cash flow, we need the income statement. So, the income statement for the year is:

Income Statement

| Sales | $\$ 30,670$ |
| :--- | ---: |
| Costs | 15,380 |
| Depreciation | $\underline{2,590}$ |
| EBIT | $\$ 12,700$ |
| Interest | $\underline{480}$ |
| Taxable income | $\$ 12,220$ |
| Taxes (35\%) | $\underline{4,277}$ |
| Net income | $\underline{\$ 7,943}$ |

Now we can calculate the operating cash flow which is:
OCF $=$ EBIT + Depreciation - Taxes
$\mathrm{OCF}=\$ 12,700+2,590-4,277=\$ 11,013$
And the cash flow from assets is:
Cash flow from assets $=$ OCF - Change in NWC - Net capital spending.
Cash flow from assets $=\$ 11,013-(-\$ 294)-3,022$
Cash flow from assets $=\$ 8,285$
d. To find the cash flow to creditors, we first need to find the net new borrowing. The net new borrowing is the difference between the ending long-term debt and the beginning long-term debt, so:

Net new borrowing $=\mathrm{LTD}_{\text {Ending }}-\mathrm{LTD}_{\text {Beginnning }}$
Net new borrowing $=\$ 6,048-5,184$
Net new borrowing $=\$ 864$
So, the cash flow to creditors is:
Cash flow to creditors $=$ Interest - Net new borrowing
Cash flow to creditors $=\$ 480-864=-\$ 384$

## SOLUTIONS B-18

The net new borrowing is also the difference between the debt issued and the debt retired. We know the amount the company issued during the year, so we can find the amount the company retired. The amount of debt retired was:

Net new borrowing $=$ Debt issued - Debt retired
$\$ 864=\$ 1,300-$ Debt retired
Debt retired $=\$ 436$
23. To construct the cash flow identity, we will begin cash flow from assets. Cash flow from assets is:

Cash flow from assets $=\mathrm{OCF}-$ Change in NWC - Net capital spending

So, the operating cash flow is:
$\mathrm{OCF}=\mathrm{EBIT}+$ Depreciation - Taxes
$\mathrm{OCF}=\$ 139,833+68,220-40,499$
$\mathrm{OCF}=\$ 167,554$

Next, we will calculate the change in net working capital which is:

Change in NWC $=\mathrm{NWC}_{\text {end }}-\mathrm{NWC}_{\text {beg }}$
Change in NWC $=\left(\mathrm{CA}_{\text {end }}-\mathrm{CL}_{\text {end }}\right)-\left(\mathrm{CA}_{\text {beg }}-\mathrm{CL}_{\text {beg }}\right)$
Change in NWC $=(\$ 72,700-33,723)-(\$ 57,634-30,015)$
Change in NWC $=\$ 11,358$

Now, we can calculate the capital spending. The capital spending is:
Net capital spending $=\mathrm{NFA}_{\text {end }}-\mathrm{NFA}_{\text {beg }}+$ Depreciation
Net capital spending $=\$ 507,888-430,533+68,220$
Net capital spending $=\$ 145,575$

Now, we have the cash flow from assets, which is:

Cash flow from assets $=\mathrm{OCF}-$ Change in NWC - Net capital spending
Cash flow from assets $=\$ 167,554-11,358-145,575$
Cash flow from assets $=\$ 10,621$

The company generated $\$ 10,621$ in cash from its assets. The cash flow from operations was $\$ 167,554$, and the company spent $\$ 11,358$ on net working capital and $\$ 145,575$ in fixed assets.

The cash flow to creditors is:

Cash flow to creditors $=$ Interest paid - New long-term debt
Cash flow to creditors $=$ Interest paid $-\left(\right.$ Long-term debt ${ }_{\text {end }}-$ Long-term debt $\left.{ }_{\text {beg }}\right)$
Cash flow to creditors $=\$ 24,120-(\$ 190,000-171,000)$
Cash flow to creditors $=\$ 5,120$

The cash flow to stockholders is a little trickier in this problem. First, we need to calculate the new equity sold. The equity balance increased during the year. The only way to increase the equity balance is to add addition to retained earnings or sell equity. To calculate the new equity sold, we can use the following equation:

New equity $=$ Ending equity - Beginning equity - Addition to retained earnings
New equity $=\$ 356,865-287,152-63,214$
New equity $=\$ 6,499$
What happened was the equity account increased by $\$ 69,713$. $\$ 63,214$ of this came from addition to retained earnings, so the remainder must have been the sale of new equity. Now we can calculate the cash flow to stockholders as:

Cash flow to stockholders $=$ Dividends paid - Net new equity
Cash flow to stockholders $=\$ 12,000-6,499$
Cash flow to stockholders $=\$ 5,501$
The company paid $\$ 5,120$ to creditors and $\$ 5,500$ to stockholders.
Finally, the cash flow identity is:
Cash flow from assets $=$ Cash flow to creditors + Cash flow to stockholders $\$ 10,621=\$ 5,120+\quad \$ 5,501$

The cash flow identity balances, which is what we expect.

## Challenge

24. Net capital spending $=\mathrm{NFA}_{\text {end }}-\mathrm{NFA}_{\text {beg }}+$ Depreciation

$$
\begin{aligned}
& =\left(\text { NFA }_{\text {end }}-\text { NFA }_{\text {beg }}\right)+\left(\text { Depreciation }+A D_{\text {beg }}\right)-A D_{\text {beg }} \\
& =\left(\text { NFA }_{\text {end }}-\text { NFA }_{\text {beg }}\right)+\mathrm{AD}_{\text {end }}-A D_{\text {beg }} \\
& =\left(\mathrm{NFA}_{\text {end }}+\mathrm{AD}_{\text {end }}\right)-\left(\mathrm{NFA}_{\text {beg }}+A D_{\text {beg }}\right) \\
& =\mathrm{FA}_{\text {end }}-\mathrm{FA}_{\text {beg }}
\end{aligned}
$$

25. a. The tax bubble causes average tax rates to catch up to marginal tax rates, thus eliminating the tax advantage of low marginal rates for high income corporations.
b. Taxes $=0.15(\$ 50 \mathrm{~K})+0.25(\$ 25 \mathrm{~K})+0.34(\$ 25 \mathrm{~K})+0.39(\$ 235 \mathrm{~K})=\$ 113.9 \mathrm{~K}$

Average tax rate $=\$ 113.9 \mathrm{~K} / \$ 335 \mathrm{~K}=34 \%$
The marginal tax rate on the next dollar of income is 34 percent.
For corporate taxable income levels of $\$ 335 \mathrm{~K}$ to $\$ 10 \mathrm{M}$, average tax rates are equal to marginal tax rates.

Taxes $=0.34(\$ 10 \mathrm{M})+0.35(\$ 5 \mathrm{M})+0.38(\$ 3.333 \mathrm{M})=\$ 6,416,667$
Average tax rate $=\$ 6,416,667 / \$ 18,333,334=35 \%$

## SOLUTIONS B-20

The marginal tax rate on the next dollar of income is 35 percent. For corporate taxable income levels over $\$ 18,333,334$, average tax rates are again equal to marginal tax rates.
c. At the end of the "tax bubble", the marginal tax rate on the next dollar should equal the average tax rate on all preceding dollars. Since the upper threshold of the bubble bracket is now $\$ 200,000$, the marginal tax rate on dollar $\$ 200,001$ should be 34 percent, and the total tax paid on the first $\$ 200,000$ should be $\$ 200,000(.34)$. So, we get:

$$
\begin{array}{ll}
\text { Taxes } & =0.34(\$ 200 \mathrm{~K})=\$ 68 \mathrm{~K}=0.15(\$ 50 \mathrm{~K})+0.25(\$ 25 \mathrm{~K})+0.34(\$ 25 \mathrm{~K})+\mathrm{X}(\$ 100 \mathrm{~K}) \\
\mathrm{X}(\$ 100 \mathrm{~K}) & =\$ 68 \mathrm{~K}-22.25 \mathrm{~K}=\$ 45.75 \mathrm{~K} \\
\mathrm{X} & =\$ 45.75 \mathrm{~K} / \$ 100 \mathrm{~K} \\
\mathrm{X} & =45.75 \%
\end{array}
$$

## CHAPTER 3

## WORKING WITH FINANCIAL STATEMENTS

## Answers to Concepts Review and Critical Thinking Questions

1. a. If inventory is purchased with cash, then there is no change in the current ratio. If inventory is purchased on credit, then there is a decrease in the current ratio if it was initially greater than 1.0.
b. Reducing accounts payable with cash increases the current ratio if it was initially greater than 1.0.
c. Reducing short-term debt with cash increases the current ratio if it was initially greater than 1.0.
d. As long-term debt approaches maturity, the principal repayment and the remaining interest expense become current liabilities. Thus, if debt is paid off with cash, the current ratio increases if it was initially greater than 1.0. If the debt has not yet become a current liability, then paying it off will reduce the current ratio since current liabilities are not affected.
$e$. Reduction of accounts receivables and an increase in cash leaves the current ratio unchanged.
$f$. Inventory sold at cost reduces inventory and raises cash, so the current ratio is unchanged.
g. Inventory sold for a profit raises cash in excess of the inventory recorded at cost, so the current ratio increases.
2. The firm has increased inventory relative to other current assets; therefore, assuming current liability levels remain mostly unchanged, liquidity has potentially decreased.
3. A current ratio of 0.50 means that the firm has twice as much in current liabilities as it does in current assets; the firm potentially has poor liquidity. If pressed by its short-term creditors and suppliers for immediate payment, the firm might have a difficult time meeting its obligations. A current ratio of 1.50 means the firm has $50 \%$ more current assets than it does current liabilities. This probably represents an improvement in liquidity; short-term obligations can generally be met completely with a safety factor built in. A current ratio of 15.0 , however, might be excessive. Any excess funds sitting in current assets generally earn little or no return. These excess funds might be put to better use by investing in productive long-term assets or distributing the funds to shareholders.
4. $a$. Quick ratio provides a measure of the short-term liquidity of the firm, after removing the effects of inventory, generally the least liquid of the firm's current assets.
b. Cash ratio represents the ability of the firm to completely pay off its current liabilities balance with its most liquid asset (cash).

## B-22 SOLUTIONS

c. The capital intensity ratio tells us the dollar amount investment in assets needed to generate one dollar in sales.
d. Total asset turnover measures how much in sales is generated by each dollar of firm assets.
e. Equity multiplier represents the degree of leverage for an equity investor of the firm; it measures the dollar worth of firm assets each equity dollar has a claim to.
f. Long-term debt ratio measures the percentage of total firm capitalization funded by long-term debt.
g. Times interest earned ratio provides a relative measure of how well the firm's operating earnings can cover current interest obligations.
h. Profit margin is the accounting measure of bottom-line profit per dollar of sales.
i. Return on assets is a measure of bottom-line profit per dollar of total assets.
$j$. Return on equity is a measure of bottom-line profit per dollar of equity.
k. Price-earnings ratio reflects how much value per share the market places on a dollar of accounting earnings for a firm.
5. Common size financial statements express all balance sheet accounts as a percentage of total assets and all income statement accounts as a percentage of total sales. Using these percentage values rather than nominal dollar values facilitates comparisons between firms of different size or business type.
6. Peer group analysis involves comparing the financial ratios and operating performance of a particular firm to a set of peer group firms in the same industry or line of business. Comparing a firm to its peers allows the financial manager to evaluate whether some aspects of the firm's operations, finances, or investment activities are out of line with the norm, thereby providing some guidance on appropriate actions to take to adjust these ratios, if appropriate. An aspirant group would be a set of firms whose performance the company in question would like to emulate. The financial manager often uses the financial ratios of aspirant groups as the target ratios for his or her firm; some managers are evaluated by how well they match the performance of an identified aspirant group.
7. Return on equity is probably the most important accounting ratio that measures the bottom-line performance of the firm with respect to the equity shareholders. The Du Pont identity emphasizes the role of a firm's profitability, asset utilization efficiency, and financial leverage in achieving a ROE figure. For example, a firm with ROE of $20 \%$ would seem to be doing well, but this figure may be misleading if it were a marginally profitable (low profit margin) and highly levered (high equity multiplier). If the firm's margins were to erode slightly, the ROE would be heavily impacted.
8. The book-to-bill ratio is intended to measure whether demand is growing or falling. It is closely followed because it is a barometer for the entire high-tech industry where levels of revenues and earnings have been relatively volatile.
9. If a company is growing by opening new stores, then presumably total revenues would be rising. Comparing total sales at two different points in time might be misleading. Same-store sales control for this by only looking at revenues of stores open within a specific period.
10. $a$. For an electric utility such as Con Ed, expressing costs on a per kilowatt hour basis would be a way comparing costs with other utilities of different sizes.
$b$. For a retailer such as JC Penney, expressing sales on a per square foot basis would be useful in comparing revenue production against other retailers.
c. For an airline such as Delta, expressing costs on a per passenger mile basis allows for comparisons with other airlines by examining how much it costs to fly one passenger one mile.
d. For an on-line service such as AOL, using a per call basis for costs would allow for comparisons with smaller services. A per subscriber basis would also make sense.
e. For a hospital such as Holy Cross, revenues and costs expressed on a per bed basis would be useful.
f. For a college textbook publisher such as McGraw-Hill/Irwin, the leading publisher of finance textbooks for the college market, the obvious standardization would be per book sold.
11. As with any ratio analysis, the ratios themselves do not necessarily indicate a problem, but simply indicate that something is different and it is up to us to determine if a problem exists. If the cost of goods sold as a percentage of sales is increasing, we would expect that EBIT as a percentage of sales would decrease, all else constant. An increase in the cost of goods sold as a percentage of sales occurs because the cost of raw materials or other inventory is increasing at a faster rate than the sales price.

This is may be a bad sign since the contribution of each sales dollar to net income and cash flow is lower. However, when a new product, for example, the HDTV, enters the market, the price of one unit will often be high relative to the cost of goods sold per unit, and demand, therefore sales, initially small. As the product market becomes more developed, price of the product generally drops, and sales increase as more competition enters the market. In this case, the increase in cost of goods sold as a percentage of sales is to be expected. The maker or seller expects to boost sales at a faster rate than its cost of goods sold increases. In this case, a good practice would be to examine the common-size income statements to see if this is an industry-wide occurrence.
12. If we assume that the cause is negative, the two reasons for the trend of increasing cost of goods sold as a percentage of sales are that costs are becoming too high or the sales price is not increasing fast enough. If the cause is an increase in the cost of goods sold, the manager should look at possible actions to control costs. If costs can be lowered by seeking lower cost suppliers of similar or higher quality, the cost of goods sold as a percentage of sales should decrease. Another alternative is to increase the sales price to cover the increase in the cost of goods sold. Depending on the industry, this may be difficult or impossible. For example, if the company sells most of its products under a long-term contract that has a fixed price, it may not be able to increase the sales price and will be forced to look for other cost-cutting possibilities. Additionally, if the market is competitive, the company might also be unable to increase the sales price.

## B-24 SOLUTIONS

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. To find the current assets, we must use the net working capital equation. Doing so, we find:
```
NWC = Current assets - Current liabilities
\(\$ 1,350=\) Current assets \(-\$ 4,290\)
Current assets \(=\$ 5,640\)
```

Now, use this number to calculate the current ratio and the quick ratio. The current ratio is:

```
Current ratio = Current assets / Current liabilities
Current ratio = $5,640 / $4,290
Current ratio = 1.31 times
```

And the quick ratio is:
Quick ratio $=($ Current assets - Inventory $) /$ Current liabilities
Quick ratio $=(\$ 5,640-1,820) / \$ 4,290$
Quick ratio $=0.89$ times
2. To find the return on assets and return on equity, we need net income. We can calculate the net income using the profit margin. Doing so, we find the net income is:

Profit margin $=$ Net income $/$ Sales
$.08=$ Net income $/ \$ 27,000,000$
Net income $=\$ 2,160,000$
Now we can calculate the return on assets as:
ROA $=$ Net income $/$ Total assets
ROA $=\$ 2,160,000 / \$ 99,000,000$
ROA $=0.1137$ or $11.37 \%$
We do not have the equity for the company, but we know that equity must be equal to total assets minus total debt, so the ROE is:

ROE $=$ Net income $/($ Total assets - Total debt $)$
ROE $=\$ 2,160,000 /(\$ 19,000,000-6,400,000)$
ROE $=0.1717$ or $17.14 \%$
3. The receivables turnover for the company was:

Receivables turnover $=$ Credit sales $/$ Receivables
Receivables turnover $=\$ 5,871,650 / \$ 645,382$
Receivables turnover $=9.10$ times
Using the receivables turnover, we can calculate the day's sales in receivables as:
Days' sales in receivables $=365$ days $/$ Receivables turnover
Days' sales in receivables $=365$ days $/ 9.10$
Days' sales in receivables $=40.12$ days
The average collection period, which is the same as the day's sales in receivables, was 40.12 days.
4. The inventory turnover for the company was:

Inventory turnover $=$ COGS $/$ Inventory
Inventory turnover $=\$ 8,493,825 / \$ 743,186$
Inventory turnover $=11.43$ times
Using the inventory turnover, we can calculate the days' sales in inventory as:
Days' sales in inventory $=365$ days $/$ Inventory turnover
Days' sales in inventory $=365$ days / 11.43
Days' sales in inventory $=31.94$ days
On average, a unit of inventory sat on the shelf 31.94 days before it was sold.
5. To find the debt-equity ratio using the total debt ratio, we need to rearrange the total debt ratio equation. We must realize that the total assets are equal to total debt plus total equity. Doing so, we find:

Total debt ratio $=$ Total debt $/$ Total assets
$0.70=$ Total debt $/($ Total debt + Total equity $)$
$0.30($ Total debt $)=0.70($ Total equity $)$
Total debt $/$ Total equity $=0.70 / 0.30$
Debt-equity ratio $=2.33$
And the equity multiplier is one plus the debt-equity ratio, so:
Equity multiplier $=1+\mathrm{D} / \mathrm{E}$
Equity multiplier $=1+2.33$
Equity multiplier $=3.33$

## B-26 SOLUTIONS

6. We need to calculate the net income before we calculate the earnings per share. The sum of dividends and addition to retained earnings must equal net income, so net income must have been:

Net income $=$ Addition to retained earnings + Dividends
Net income $=\$ 530,000+190,000$
Net income $=\$ 720,000$
So, the earnings per share were:
EPS $=$ Net income $/$ Shares outstanding
EPS $=\$ 720,000 / 570,000$
$\mathrm{EPS}=\$ 1.26$ per share
The dividends per share were:
Dividends per share $=$ Total dividends $/$ Shares outstanding
Dividends per share $=\$ 190,000 / 570,000$
Dividends per share $=\$ 0.33$ per share
The book value per share was:
Book value per share $=$ Total equity $/$ Shares outstanding
Book value per share $=\$ 6,800,000 / 570,000$
Book value per share $=\$ 11.93$ per share

The market-to-book ratio is:

Market-to-book ratio $=$ Share price $/$ Book value per share
Market-to-book ratio $=\$ 39 / \$ 11.93$
Market-to-book ratio $=3.27$ times
The $\mathrm{P} / \mathrm{E}$ ratio is:
$\mathrm{P} / \mathrm{E}$ ratio $=$ Share price $/ \mathrm{EPS}$
$\mathrm{P} / \mathrm{E}$ ratio $=\$ 39 / \$ 1.26$
$\mathrm{P} / \mathrm{E}$ ratio $=30.88$ times

Sales per share are:

Sales per share $=$ Total sales $/$ Shares outstanding
Sales per share $=\$ 16,000,000 / 570,000$
Sales per share $=\$ 28.07$

The $\mathrm{P} / \mathrm{S}$ ratio is:
$\mathrm{P} / \mathrm{S}$ ratio $=$ Share price $/$ Sales per share
$\mathrm{P} / \mathrm{S}$ ratio $=\$ 39 / \$ 28.07$
$\mathrm{P} / \mathrm{S}$ ratio $=1.39$ times
7. With the information given, we must use the Du Pont identity to calculate return on equity. Doing so, we find:

ROE $=($ Profit margin)(Total asset turnover)(Equity multiplier)
ROE $=(.08)(1.32)(1.60)$
ROE $=0.1690$ or $16.90 \%$
8. We can use the Du Pont identity and solve for the equity multiplier. With the equity multiplier we can find the debt-equity ratio. Doing so we find:

ROE $=($ Profit margin)(Total asset turnover)(Equity multiplier)
$0.1570=(0.10)(1.35)($ Equity multiplier)
Equity multiplier $=1.16$
Now, using the equation for the equity multiplier, we get:
Equity multiplier $=1+$ Debt-equity ratio
$1.16=1+$ Debt-equity ratio
Debt-equity ratio $=0.16$
9. To find the days' sales in payables, we first need to find the payables turnover. The payables turnover was:

Payables turnover $=$ Cost of goods sold $/$ Payables balance
Payables turnover $=\$ 48,813 / \$ 11,816$
Payables turnover $=4.13$ times
Now, we can use the payables turnover to find the days' sales in payables as:
Days' sales in payables $=365$ days $/$ Payables turnover
Days' sales in payables $=365$ days $/ 4.13$
Days' sales in payables $=88.35$ days
The company left its bills to suppliers outstanding for 88.35 days on average. A large value for this ratio could imply that either (1) the company is having liquidity problems, making it difficult to pay off its short-term obligations, or (2) that the company has successfully negotiated lenient credit terms from its suppliers.
10. With the information provided, we need to calculate the return on equity using an extended return on equity equation. We first need to find the equity multiplier which is:

Equity multiplier $=1+$ Debt-equity ratio
Equity multiplier $=1+0.80$
Equity multiplier $=1.80$

## B-28 SOLUTIONS

Now we can calculate the return on equity as:
ROE $=($ ROA $)($ Equity multiplier $)$
ROE $=0.089(1.80)$
ROE $=0.1602$ or $16.02 \%$
The return on equity equation we used was an abbreviated version of the Du Pont identity. If we multiply the profit margin and total asset turnover ratios from the Du Pont identity, we get:
$($ Net income $/$ Sales $)($ Sales $/$ Total assets $)=$ Net income $/$ Total assets $=$ ROA
With the return on equity, we can calculate the net income as:
ROE $=$ Net income / Total equity
$0.1602=$ Net income / \$590,000
Net income $=\$ 94,518$
11. To find the internal growth rate, we need the plowback, or retention, ratio. The plowback ratio is:
$\mathrm{b}=1-0.20$
$\mathrm{b}=0.80$
Now, we can use the internal growth rate equation to find:
Internal growth rate $=[(\mathrm{ROA})(\mathrm{b})] /[1-(\mathrm{ROA})(\mathrm{b})]$
Internal growth rate $=[0.11(0.80)] /[1-0.11(0.80)]$
Internal growth rate $=0.0965$ or $9.65 \%$
12. To find the internal growth rate we need the plowback, or retention, ratio. The plowback ratio is:
$\mathrm{b}=1-0.25$
$\mathrm{b}=0.75$
Now, we can use the sustainable growth rate equation to find:
Sustainable growth rate $=[(\operatorname{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
Sustainable growth rate $=[0.142(0.75)] /[1-0.142(0.75)]$
Sustainable growth rate $=0.1192$ or $11.92 \%$
13. We need the return on equity to calculate the sustainable growth rate. To calculate return on equity, we need to realize that the total asset turnover is the inverse of the capital intensity ratio and the equity multiplier is one plus the debt-equity ratio. So, the return on equity is:

ROE $=($ Profit margin)(Total asset turnover)(Equity multiplier)
ROE $=(0.74)(1 / 0.55)(1+0.30)$
ROE $=0.1749$ or $17.49 \%$

Next we need the plowback ratio. The payout ratio is one minus the payout ratio. We can calculate the payout ratio as the dividends divided by net income, so the plowback ratio is:
$\mathrm{b}=1-(\$ 25,000 / \$ 70,000)$
$\mathrm{b}=0.64$

Now we can use the sustainable growth rate equation to find:

Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
Sustainable growth rate $=[0.1749(0.64)] /[1-0.1749(0.64)]$
Sustainable growth rate $=0.1267$ or $12.67 \%$
14. We need the return on equity to calculate the sustainable growth rate. Using the Du Pont identity, the return on equity is:

ROE $=($ Profit margin $)($ Total asset turnover $)($ Equity multiplier $)$
ROE $=(.083)(1.75)(1.85)$
ROE $=.2687$ or $26.87 \%$

To find the sustainable growth rate, we need the plowback, or retention, ratio. The plowback ratio is:
$\mathrm{b}=1-.40$
$b=.60$

Now, we can use the sustainable growth rate equation to find:

Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
Sustainable growth rate $=[.2687(.60)] /[1-.2687(.60)]$
Sustainable growth rate $=.1922$ or $19.22 \%$
15. To calculate the common-size balance sheet, we divide each asset account by total assets, and each liability and equity account by total liabilities and equity. For example, the common-size cash percentage for 2007 is:

Cash percentage $=$ Cash $/$ Total assets
Cash percentage $=\$ 18,288 / \$ 748,879$
Cash percentage $=0.0244$ or $2.44 \%$

## B-30 SOLUTIONS

Repeating this procedure for each account, we get:

|  | 2007 |  | 2008 |  |
| :---: | :---: | :---: | :---: | :---: |
| Assets |  |  |  |  |
| Current assets |  |  |  |  |
| Cash | \$18,288 | 2.44\% | \$22,455 | 2.86\% |
| Accounts receivable | 44,062 | 5.88\% | 55,457 | 7.07\% |
| Inventory | 104,339 | 13.93\% | 144,696 | 18.44\% |
| Total | \$166,689 | 22.26\% | \$222,608 | 28.37\% |
| Fixed assets |  |  |  |  |
| Net plant and equipment | 582,190 | 77.74\% | 561,988 | 71.63\% |
| Total assets | \$748,879 | 100\% | \$784,596 | 100\% |

## Liabilities and owners' equity

| Current liabilities |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Accounts payable | \$149,940 | 20.02\% | \$144,722 | 18.45\% |
| Notes payable | 69,246 | 9.25\% | 101,134 | 12.89\% |
| Total | \$219,186 | 29.27\% | \$245,856 | 31.34\% |
| Long-term debt | 190,000 | 25.37\% | 131,250 | 16.73\% |
| Owners' equity <br> Common stock and paid-in surplus | \$160,000 | 21.37\% | \$160,000 | 20.39\% |
| Accumulated retained earnings | 179,693 | 23.99\% | 247,490 | 31.54\% |
| Total | \$339,693 | 45.36\% | \$407,490 | 51.94\% |
| Total liabilities and owners' equity | \$748,879 | 100\% | \$784,596 | 100\% |

16. $a$. The current ratio is calculated as:

Current ratio $=$ Current assets $/$ Current liabilities
Current ratio ${ }_{2007}=\$ 166,869 / \$ 219,186$
Current ratio $2007=0.76$ times
Current ratio ${ }_{2008}=\$ 222,608 / \$ 245,856$
Current ratio ${ }_{2008}=0.91$ times
b. The quick ratio is calculated as:

Quick ratio $=($ Current assets - Inventory $) /$ Current liabilities
Quick ratio ${ }_{2007}=(\$ 166,689-104,339) / \$ 219,186$
Quick ratio ${ }_{2007}=0.28$ times

Quick ratio ${ }_{2008}=(\$ 222,608-144,696) / \$ 245,856$
Quick ratio ${ }_{2008}=0.32$ times
c. The cash ratio is calculated as:

Cash ratio $=$ Cash $/$ Current liabilities
Cash ratio ${ }_{2007}=\$ 18,288 / \$ 219,186$
Cash ratio ${ }_{2007}=0.08$ times

Cash ratio ${ }_{2008}=\$ 22,455 / \$ 245,856$
Cash ratio ${ }_{2008}=0.09$ times
d. The debt-equity ratio is calculated as:

Debt-equity ratio $=$ Total debt $/$ Total equity
Debt-equity ratio $=($ Current liabilities + Long-term debt $) /$ Total equity
Debt-equity ratio ${ }_{2007}=(\$ 219,186+190,000) / \$ 339,693$
Debt-equity ratio $_{2007}=1.20$
Debt-equity ratio ${ }_{2008}=(\$ 245,856+131,250) / \$ 407,490$
Debt-equity ratio ${ }_{2008}=0.93$
And the equity multiplier is:
Equity multiplier $=1+$ Debt-equity ratio
Equity multiplier ${ }_{2007}=1+1.20$
Equity multiplier $_{2007}=2.20$

Equity multiplier ${ }_{2008}=1+0.93$
Equity multiplier $_{2008}=1.93$
e. The total debt ratio is calculated as:

Total debt ratio $=$ Total debt $/$ Total assets
Total debt ratio $=($ Current liabilities + Long-term debt $) /$ Total assets

Total debt ratio $_{2007}=(\$ 219,186+190,000) / \$ 748,879$
Total debt ratio $_{2007}=0.55$
Total debt ratio $_{2007}=(\$ 245,856+131,250) / \$ 784,596$
Total debt ratio $_{2007}=0.48$

## B-32 SOLUTIONS

17. Using the Du Pont identity to calculate ROE, we get:

ROE $=($ Profit margin $)($ Total asset turnover $)($ Equity multiplier $)$
ROE $=($ Net income $/$ Sales $)($ Sales $/$ Total assets)(Total asset $/$ Total equity)
ROE $=(\$ 132,186 / \$ 2,678,461)(\$ 2,678,461 / \$ 784,596)(\$ 784,596 / \$ 407,490)$
ROE $=0.3244$ or $32.44 \%$
18. One equation to calculate ROA is:
$\mathrm{ROA}=($ Profit margin $)($ Total asset turnover $)$

We can solve this equation to find total asset turnover as:
$0.12=0.07$ (Total asset turnover)
Total asset turnover $=1.71$ times

Now, solve the ROE equation to find the equity multiplier which is:
ROE $=($ ROA $)($ Equity multiplier $)$
$0.17=0.12$ (Equity multiplier)
Equity multiplier $=1.42$ times
19. To calculate the ROA, we first need to find the net income. Using the profit margin equation, we find:

Profit margin $=$ Net income $/$ Sales
$0.075=$ Net income / \$26,000,000
Net income $=\$ 1,950,000$

Now we can calculate ROA as:

ROA $=$ Net income / Total assets
ROA $=\$ 1,950,000 / \$ 19,000,000$
ROA $=0.1026$ or $10.26 \%$
20. To calculate the internal growth rate, we need to find the ROA and the plowback ratio. The ROA for the company is:

ROA $=$ Net income $/$ Total assets
ROA $=\$ 11,687 / \$ 86,900$
$\mathrm{ROA}=0.1345$ or $13.45 \%$

And the plowback ratio is:
$\mathrm{b}=1-.40$
$\mathrm{b}=.60$

Now, we can use the internal growth rate equation to find:
Internal growth rate $=[(\mathrm{ROA})(\mathrm{b})] /[1-(\mathrm{ROA})(\mathrm{b})]$
Internal growth rate $=[0.1345(0.60)] /[1-0.1345(.60)]$
Internal growth rate $=0.0878$ or $8.78 \%$
21. To calculate the sustainable growth rate, we need to find the ROE and the plowback ratio. The ROE for the company is:

ROE $=$ Net income $/$ Equity
ROE $=\$ 11,687 / \$ 58,300$
ROE $=0.2005$ or $20.05 \%$
Using the sustainable growth rate, we calculated in the precious problem, we find the sustainable growth rate is:

Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
Sustainable growth rate $=[(0.2005)(0.60)] /[1-(0.2005)(0.60)]$
Sustainable growth rate $=0.1367$ or $13.67 \%$
22. The total asset turnover is:

Total asset turnover $=$ Sales $/$ Total assets
Total asset turnover $=\$ 17,000,000 / \$ 7,000,000=2.43$ times
If the new total asset turnover is 2.75 , we can use the total asset turnover equation to solve for the necessary sales level. The new sales level will be:

Total asset turnover $=$ Sales $/$ Total assets
2.75 = Sales / \$7,000,000

Sales $=\$ 19,250,000$
23. To find the ROE, we need the equity balance. Since we have the total debt, if we can find the total assets we can calculate the equity. Using the total debt ratio, we find total assets as:

Debt ratio $=$ Total debt $/$ Total assets
$0.70=\$ 265,000 /$ Total assets
Total assets $=\$ 378,571$
Total liabilities and equity is equal to total assets. Using this relationship, we find:
Total liabilities and equity $=$ Total debt + Total equity
$\$ 378,571=\$ 265,000+$ Total equity
Total equity $=\$ 113,571$

## B-34 SOLUTIONS

Now, we can calculate the ROE as:
ROE $=$ Net income / Total equity
ROE $=\$ 24,850 / \$ 113,571$
ROE $=0.2188$ or $21.88 \%$
24. The earnings per share are:

```
EPS = Net income / Shares
EPS = $5,150,000 / 4,100,000
EPS = $1.26
```

The price-earnings ratio is:
P/E = Price / EPS
$\mathrm{P} / \mathrm{E}=\$ 41 / \$ 1.26$
$\mathrm{P} / \mathrm{E}=32.64$
The sales per share are:
Sales per share $=$ Sales $/$ Shares
Sales per share $=\$ 39,000,000 / 4,100,000$
Sales per share $=\$ 9.51$
The price-sales ratio is:
P/S = Price / Sales per share
P/S = \$41 / \$9.51
$\mathrm{P} / \mathrm{S}=4.31$
The book value per share is:
Book value per share $=$ Book value of equity $/$ Shares
Book value per share $=\$ 21,580,000 / 4,100,000$
Book value per share $=\$ 5.26$ per share
And the market-to-book ratio is:
Market-to-book $=$ Market value per share $/$ Book value per share
Market-to-book $=\$ 41 / \$ 5.26$
Market-to-book $=7.79$
25. To find the profit margin, we need the net income and sales. We can use the total asset turnover to find the sales and the return on assets to find the net income. Beginning with the total asset turnover, we find sales are:

Total asset turnover $=$ Sales $/$ Total assets
$2.10=$ Sales / \$10,500,000
Sales $=\$ 22,050,000$
And the net income is:
ROA $=$ Net income $/$ Total assets
$0.13=$ Net income $/ \$ 10,500,000$
Net income $=\$ 1,365,000$
Now we can find the profit margin which is:
Profit margin $=$ Net income $/$ Sales
Profit margin $=\$ 1,365,000 / \$ 22,050,000$
Profit margin $=0.0619$ or $6.19 \%$

## Intermediate

26. We can rearrange the Du Pont identity to calculate the profit margin. So, we need the equity multiplier and the total asset turnover. The equity multiplier is:

Equity multiplier $=1+$ Debt-equity ratio
Equity multiplier $=1+.25$
Equity multiplier $=1.25$
And the total asset turnover is:
Total asset turnover $=$ Sales $/$ Total assets
Total asset turnover $=\$ 9,980 / \$ 3,140$
Total asset turnover $=3.18$ times
Now, we can use the Du Pont identity to find total sales as:
ROE $=($ Profit margin $)($ Total asset turnover $)($ Equity multiplier $)$
$0.16=(\mathrm{PM})(3.18)(1.25)$
Profit margin $=0.403$ or $4.03 \%$
Rearranging the profit margin ratio, we can find the net income which is:
Profit margin $=$ Net income $/$ Sales
$0.0403=$ Net income / \$9,980
Net income $=\$ 401.92$

## B-36 SOLUTIONS

27. This is a multi-step problem in which we need to calculate several ratios to find the fixed assets. If we know total assets and current assets, we can calculate the fixed assets. Using the current ratio to find the current assets, we get:

Current ratio $=$ Current assets $/$ Current liabilities
$1.30=$ Current assets $/ \$ 900$
Current assets $=\$ 1,170.00$
Now, we are going to use the profit margin to find the net income and use the net income to find the equity. Doing so, we get:

Profit margin $=$ Net income $/$ Sales
$0.09=$ Net income / \$6,590
Net income $=\$ 593.10$
And using this net income figure in the return on equity equation to find the equity, we get:
ROE $=$ Net income $/$ Total equity
$0.16=\$ 593.10 /$ Total equity
Total equity $=\$ 3,706.88$
Now, we can use the long-term debt ratio to find the total long-term debt. The equation is:
Long-term debt ratio $=$ Long-term debt $/($ Long-term debt + Total equity $)$
Inverting both sides we get:
$1 /$ Long-term debt $=1+($ Total equity $/$ Long-term debt $)$
$1 / 0.60=1+($ Total equity / Long-term debt)
Total equity $/$ Long-term debt $=0.667$
$\$ 1,907.03 /$ Long-term debt $=0.667$
Long-term debt $=\$ 5,560.31$
Now, we can calculate the total debt as:
Total debt $=$ Current liabilities + Long-term debt
Total debt $=\$ 900+5,560.31$
Total debt $=\$ 6,460.31$
This allows us to calculate the total assets as:
Total assets $=$ Total debt + Total equity
Total assets $=\$ 6,460.31+3,706.81$
Total assets $=\$ 10,167.19$

Finally, we can calculate the net fixed assets as:
Net fixed assets $=$ Total assets - Current assets
Net fixed assets $=\$ 10,167.19-1,170$
Net fixed assets $=\$ 8,997.19$
28. The child's profit margin is:

Profit margin $=$ Net income $/$ Sales
Profit margin $=\$ 1 / \$ 25$
Profit margin $=0.04$ or $4 \%$
And the store's profit margin is:
Profit margin $=$ Net income $/$ Sales
Profit margin $=\$ 13,200,000 / \$ 660,000,000$
Profit margin $=0.02$ or $2 \%$
The advertisement is referring to the store's profit margin, but a more appropriate earnings measure for the firm's owners is the return on equity. The store's return on equity is:

ROE $=$ Net income $/$ Total equity
ROE $=$ Net income $/($ Total assets - Total debt $)$
ROE $=\$ 13,200,000 /(\$ 280,000,000-151,500,000)$
ROE $=0.1027$ or $10.27 \%$
29. To calculate the profit margin, we first need to calculate the sales. Using the days' sales in receivables, we find the receivables turnover is:

Days' sales in receivables $=365$ days $/$ Receivables turnover
29.70 days $=365$ days $/$ Receivables turnover

Receivables turnover $=12.29$ times
Now, we can use the receivables turnover to calculate the sales as:
Receivables turnover $=$ Sales $/$ Receivables
$12.29=$ Sales / \$138,600
Sales $=\$ 1,703,333$
So, the profit margin is:
Profit margin $=$ Net income $/$ Sales
Profit margin $=\$ 132,500 / \$ 1,703,333$
Profit margin $=0.0778$ or $7.78 \%$

## B-38 SOLUTIONS

The total asset turnover is:
Total asset turnover $=$ Sales $/$ Total assets
Total asset turnover $=\$ 1,703,333 / \$ 820,000$
Total asset turnover $=2.08$ times
We need to use the Du Pont identity to calculate the return on equity. Using this relationship, we get:

ROE $=($ Profit margin)(Total asset turnover) $(1+$ Debt-equity ratio $)$
ROE $=(0.778)(2.08)(1+0.60)$
ROE $=0.2585$ or $25.85 \%$
30. Here, we need to work the income statement backward to find the EBIT. Starting at the bottom of the income statement, we know that the taxes are the taxable income times the tax rate. The net income is the taxable income minus taxes. Rearranging this equation, we get:

Net income $=$ Taxable income $-\left(\mathrm{t}_{\mathrm{c}}\right)$ (Taxable income)
Net income $=\left(1-t_{\mathrm{C}}\right)($ Taxable income $)$
Using this relationship we find the taxable income is:
Net income $=\left(1-\mathrm{t}_{\mathrm{c}}\right)($ Taxable income $)$
$\$ 10,508=(1-.34)$ (Taxable income)
Taxable income = \$15,921.21
Now, we can calculate the EBIT as:
Taxable income $=$ EBIT - Interest
\$15,921.21 = EBIT - \$3,685
EBIT $=\$ 19,606.21$
So, the cash coverage ratio is:
Cash coverage ratio $=($ EBIT + Depreciation expense $) /$ Interest
Cash coverage ratio $=(\$ 19,606.21+4,382) / \$ 3,685$
Cash coverage ratio $=6.51$ times
31. To find the times interest earned, we need the EBIT and interest expense. EBIT is sales minus costs minus depreciation, so:

EBIT $=$ Sales - Costs - Depreciation
EBIT $=\$ 378,000-95,400-47,000$
EBIT $=\$ 235,600$

Now, we need the interest expense. We know the EBIT, so if we find the taxable income (EBT), the difference between these two is the interest expense. To find EBT, we must work backward through the income statement. We need total dividends paid. We can use the dividends per share equation to find the total dividends. Dong so, we find:

DPS = Dividends / Shares
\$1.40 = Dividends / 20,000
Dividends $=\$ 28,000$
Net income is the sum of dividends and addition to retained earnings, so:
Net income $=$ Dividends + Addition to retained earnings
Net income $=\$ 28,000+48,750$
Net income $=\$ 76,750$
We know that the taxes are the taxable income times the tax rate. The net income is the taxable income minus taxes. Rearranging this equation, we get:

Net income $=$ Taxable income $-\left(\mathrm{t}_{\mathrm{C}}\right)(\mathrm{EBT})$
Net income $=\left(1-\mathrm{t}_{\mathrm{C}}\right)(\mathrm{EBT})$
$\$ 76,750=(1-.34)(E B T)$
EBT $=\$ 116,288$
Now, we can use the income statement relationship:

```
EBT \(=\) EBIT - Interest
\$116,288 = \$235,600 - Interest
Interest \(=\$ 119,312\)
```

So, the times interest earned ratio is:
Times interest earned $=$ EBIT $/$ Interest
Times interest earned $=\$ 235,600 / \$ 119,312$
Times interest earned $=1.97$ times
32. To find the return on equity, we need the net income and total equity. We can use the total debt ratio to find the total assets as:

Total debt ratio $=$ Total debt $/$ Total assets
$0.30=\$ 648,000 /$ Total assets
Total assets $=\$ 2,160,000$
Using the balance sheet relationship that total assets is equal to total liabilities and equity, we find the total equity is:

Total assets $=$ Total debt + Equity
$\$ 2,160,000=\$ 648,000+$ Equity
Equity $=\$ 1,512,000$

## B-40 SOLUTIONS

We have the return on equity and the equity. We can use the return on equity equation to find net income is:

ROE $=$ Net income / Equity
$0.1650=$ Net income $/ \$ 1,512,000$
Net income $=\$ 249,480$

We have all the information necessary to calculate the ROA, Doing so, we find the ROA is:

```
ROA \(=\) Net income / Total assets
ROA \(=\$ 249,480 / \$ 2,160,000\)
\(\mathrm{ROA}=0.1155\) or \(11.55 \%\)
```

33. The currency is generally irrelevant in calculating any financial ratio. The company's profit margin is:

Profit margin $=$ Net income $/$ Sales
Profit margin $=-£ 16,182 / £ 238,165$
Profit margin $=-0.0679$ or $-6.79 \%$

As long as both net income and sales are measured in the same currency, there is no problem; in fact, except for some market value ratios like EPS and BVPS, none of the financial ratios discussed in the text are measured in terms of currency. This is one reason why financial ratio analysis is widely used in international finance to compare the business operations of firms and/or divisions across national economic borders.

We can use the profit margin we previously calculated and the dollar sales to calculate the net income. Doing so, we get:

Profit margin $=$ Net income $/$ Sales
$-0.0679=$ Net income $/ \$ 454,058$
Net income $=-\$ 30,850.74$
34. Here, we need to calculate several ratios given the financial statements. The ratios are:

## Short-term solvency ratios:

Current ratio $=$ Current assets $/$ Current liabilities

Current ratio $_{2007}=\$ 14,626 / \$ 3,375$
Current ratio ${ }_{2007}=4.33$ times
Current ratio $_{2008}=\$ 16,536 / \$ 3,714$
Current ratio $2008=4.45$ times
Quick ratio $=($ Current assets - Inventory $) /$ Current liabilities
Quick ratio $_{2007}=(\$ 14,626-8,856) / \$ 3,375$
Quick ratio $2007=1.71$ times

Quick ratio ${ }_{2008}=(\$ 16,536-9,873) / \$ 3,714$
Quick ratio ${ }_{2008}=1.79$ times

Cash ratio $=$ Cash $/$ Current liabilities

Cash ratio ${ }_{2007}=\$ 2,351 / \$ 3,375$
Cash ratio ${ }_{2007}=0.70$ times

Cash ratio $_{2008}=\$ 2,505 / \$ 3,714$
Cash ratio ${ }_{2008}=0.67$ times

Asset utilization ratios:

Total asset turnover $=$ Sales $/$ Total assets
Total asset turnover $=\$ 124,380 / \$ 68,657$
Total asset turnover $=1.81$ times

Inventory turnover $=$ COGS $/$ Inventory
Inventory turnover $=\$ 64,805 / \$ 9,873$
Inventory turnover $=6.56$ times

Receivables turnover $=$ Sales $/$ Receivables
Receivables turnover $=\$ 124,380 / \$ 4,158$
Receivables turnover $=29.91$ times

Long-term solvency ratios:
Total debt ratio $=($ Current liabilities + Long-term debt $) /$ Total assets
Total debt ratio $_{2007}=(\$ 3,375+11,500) / \$ 49,560$
Total debt ratio $2007=0.30$

Total debt ratio $_{2008}=(\$ 3,714+12,500) / \$ 68,657$
Total debt ratio $_{2008}=0.24$

Debt-equity ratio $=($ Current liabilities + Long-term debt $) /$ Total equity

Debt-equity ratio ${ }_{2007}=(\$ 3,375+11,500) / \$ 34,685$
Debt-equity ratio ${ }_{2007}=0.43$

Debt-equity ratio $_{2008}=(\$ 3,714+12,500) / \$ 52,443$
Debt-equity ratio $_{2008}=0.31$

Equity multiplier $=1+\mathrm{D} /$ E ratio

Equity multiplier ${ }_{2007}=1+0.43$
Equity multiplier ${ }_{2007}=1.43$

## B-42 SOLUTIONS

Equity multiplier ${ }_{2008}=1+0.31$
Equity multiplier $_{2008}=1.31$

Times interest earned = EBIT / Interest
Times interest earned $=\$ 55,993 / \$ 980$
Times interest earned $=57.14$ times

Cash coverage ratio $=($ EBIT + Depreciation $) /$ Interest
Cash coverage ratio $=(\$ 55,993+3,582) / \$ 980$
Cash coverage ratio $=60.79$ times
Profitability ratios:

Profit margin $=$ Net income $/$ Sales
Profit margin $=\$ 35,758 / \$ 124,380$
Profit margin $=0.2875$ or $28.75 \%$

Return on assets $=$ Net income $/$ Total assets
Return on assets $=\$ 35,758 / \$ 68,657$
Return on assets $=0.5208$ or $52.08 \%$

Return on equity $=$ Net income $/$ Total equity
Return on equity $=\$ 35,758 / \$ 52,443$
Return on equity $=0.6818$ or $68.18 \%$
35. The Du Pont identity is:

ROE $=(\mathrm{PM})($ Total asset turnover $)($ Equity multiplier $)$
ROE $=($ Net income $/$ Sales $)($ Sales $/$ Total assets)(Total assets $/$ Total equity)
ROE $=(\$ 35,758 / \$ 124,380)(\$ 124,380 / \$ 68,657)(\$ 68,657 / \$ 52,443)$
ROE $=0.6818$ or $68.18 \%$
36. To find the price-earnings ratio we first need the earnings per share. The earnings per share are:

```
EPS = Net income / Shares outstanding
EPS = $35,758 / 10,000
EPS = $3.58
```

So, the price-earnings ratio is:
$\mathrm{P} / \mathrm{E}$ ratio $=$ Share price $/ \mathrm{EPS}$
$\mathrm{P} / \mathrm{E}$ ratio $=\$ 73 / \$ 3.58$
$\mathrm{P} / \mathrm{E}$ ratio $=20.42$

The sales per share are:
Sales per share $=$ Sales $/$ Shares outstanding
Sales per share $=\$ 124,380 / 10,000$
Sales per share $=\$ 12.44$
So, the price-sales ratio is:
$\mathrm{P} / \mathrm{S}$ ratio $=$ Share price $/$ Sales per share
$\mathrm{P} / \mathrm{S}$ ratio $=\$ 73 / \$ 12.44$
$\mathrm{P} / \mathrm{S}$ ratio $=5.87$
The dividends per share are:
Dividends per share $=$ Total dividends $/$ Shares outstanding
Dividends per share $=\$ 18,000 / 10,000$ shares
Dividends per share $=\$ 1.80$ per share
To find the market-to-book ratio, we first need the book value per share. The book value per share is:

Book value per share $=$ Total equity $/$ Shares outstanding
Book value per share $=\$ 52,443 / 10,000$ shares
Book value per share $=\$ 5.24$ per share
So, the market-to-book ratio is:
Market-to-book ratio $=$ Share price $/$ Book value per share
Market-to-book ratio $=\$ 73 / \$ 5.24$
Market-to-book ratio $=13.92$ times
37. The current ratio appears to be relatively high when compared to the median; however, it is below the upper quartile, meaning that at least 25 percent of firms in the industry have a higher current ratio. Overall, it does not appear that the current ratio is out of line with the industry. The total asset turnover is low when compared to the industry. In fact, the total asset turnover is in the lower quartile. This means that the company does not use assets as efficiently overall or that the company has newer assets than the industry. This would mean that the assets have not been depreciated, which would mean a higher book value and a lower total asset turnover. The debt-equity ratio is in line with the industry, between the mean and the lower quartile. The profit margin is in the upper quartile. The company may be better at controlling costs, or has a better product which enables it to charge a premium price. It could also be negative in that the company may have too high of a margin on its sales, which could reduce overall net income.
38. To find the profit margin, we can solve the Du Pont identity. First, we need to find the retention ratio. The retention ratio for the company is:
$\mathrm{b}=1-0.25$
$\mathrm{b}=0.75$

## B-44 SOLUTIONS

Now, we can use the sustainable growth rate equation to find the ROE. Doing so, we find:
Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
$0.08=[\operatorname{ROE}(0.75)] /[1-\operatorname{ROE}(0.75)]$
ROE $=0.0988$ or $9.88 \%$
Now, we can use the Du Pont identity. We are given the total asset to sales ratio, which is the inverse of the total asset turnover, and the equity multiplier is one plus the debt-equity ratio. Solving the Du Pont identity for the profit margin, we find:

ROE $=($ Profit margin)(Total asset turnover)(Equity multiplier)
$0.0988=($ Profit margin $)(1 / 1.40)(1+0.45)$
Profit margin $=0.0954$ or $9.54 \%$
39. The earnings per share is the net income divided by the shares outstanding. Since all numbers are in millions, the earnings per share for Abercrombie \& Fitch was:

EPS $=\$ 422.19 / 87.69$
EPS $=\$ 4.81$
And the earnings per share for Ann Taylor were;
EPS $=\$ 142.98 / 69.37$
EPS $=\$ 2.06$
The market-to-book ratio is the stock price divided by the book value per share. To find the book value per share, we divide the total equity by the shares outstanding. The book value per share and market-to-book ratio for Abercrombie \& Fitch was:

Book value per share $=\$ 1,405.30 / 87.69$
Book value per share $=\$ 16.03$
Market-to-book = \$80.77 / \$16.03
Market-to-book $=5.04$

And the market-to-book ratio for Ann Taylor was:
Book value per share $=\$ 1,049.91 / 69.37$
Book value per share $=\$ 15.13$
Market-to-book $=\$ 35.33 / \$ 15.13$
Market-to-book $=2.33$

And the price-earnings ratio for Abercrombie \& Fitch was:
$\mathrm{P} / \mathrm{E}=\$ 80.77 / \$ 4.81$
$\mathrm{P} / \mathrm{E}=16.78$

And for Ann Taylor, the P/E was:
$\mathrm{P} / \mathrm{E}=\$ 35.33 / \$ 2.06$
$\mathrm{P} / \mathrm{E}=17.14$
40. To find the total asset turnover, we can solve the ROA equation. First, we need to find the retention ratio. The retention ratio for the company is:
$\mathrm{b}=1-0.35$
$\mathrm{b}=0.65$

Now, we can use the internal growth rate equation to find the ROA. Doing so, we find:

Internal growth rate $=[(\mathrm{ROA})(\mathrm{b})] /[1-(\mathrm{ROA})(\mathrm{b})]$
$0.07=[\operatorname{ROA}(0.65)] /[1-\operatorname{ROA}(0.65)]$
$\mathrm{ROA}=0.1006$ or $10.06 \%$

Now, we can use the ROA equation to find the total asset turnover is:
$\mathrm{ROA}=(\mathrm{PM})(\mathrm{TAT})$
$0.1006=(0.08)$ TAT
Total asset turnover $=1.26$ times
41. To calculate the sustainable growth rate, we need to calculate the return on equity. We can use the Du Pont identity to calculate the return on equity if we can find the equity multiplier. Using the total debt ratio, we can find the debt-equity ratio is:

Total debt ratio $=$ Total debt $/$ Total assets
$0.20=$ Total debt $/$ Total assets
$1 / 0.20=$ Total assets $/$ Total debt
$1 / 0.20=($ Total debt + Total equity $) /$ Total debt
$1 / 0.20=1+$ Total equity $/$ Total debt
Total equity $/$ Total debt $=(1 / 0.20)-1$
Total debt $/$ Total equity $=1 /[(1 / 0.20)-1]$
Total debt / Total equity $=0.25$
Debt-equity ratio $=0.25$

So, the equity multiplier is:

Equity multiplier $=1+$ Debt-equity ratio
Equity multiplier $=1+0.25$
Equity multiplier $=1.25$

Using the Du Pont identity, the ROE is:

ROE $=($ Profit margin $)($ Total asset turnover $)($ Equity multiplier $)$
ROE $=(.1050)(1.40)(1.25)$
ROE $=0.1838$ or $18.38 \%$

## B-46 SOLUTIONS

To calculate the sustainable growth rate, we also need the retention ratio. The retention ratio is:
$\mathrm{b}=1-0.15$
$\mathrm{b}=0.85$
Now we can calculate the sustainable growth rate as:

```
Sustainable growth rate = [(ROE)(b)] / [1-(ROE)(b)]
Sustainable growth rate =[0.1838(0.85)]/[1-0.1838(0.85)]
Sustainable growth rate =0.1851 or 18.51%
```

And the return on assets is:
ROA $=($ Profit margin $)($ Total asset turnover $)$
ROA $=(0.1050)(1.40)$
$\mathrm{ROA}=0.1470$ or $14.70 \%$
42. To find the sustainable growth rate, we need the retention ratio and the return on equity. The payout ratio is the dividend payment divided by net income, so:
$\mathrm{b}=1-(\$ 4,500 / \$ 19,000)$
$\mathrm{b}=0.7632$
And the return on equity is:
ROE $=$ Net income / Total equity
ROE $=\$ 19,000 / \$ 72,000$
ROE $=0.2639$ or $26.39 \%$
So, the sustainable growth rate is:

```
Sustainable growth rate = [(ROE)(b)] / [1 - (ROE)(b)]
Sustainable growth rate = [0.2639(0.7632)] / [1-0.2639(0.7632)]
Sustainable growth rate = 0.2522 or 25.22%
```

The total assets of the company are equal to the total debt plus the total equity. The total assets will increase at the sustainable growth rate, so the total assets next year will be:

New total assets $=(1+$ Sustainable growth rate)(Fixed assets)
New total assets $=(1+0.2522)(\$ 72,000+49,000)$
New total assets $=\$ 151,513.04$
We can find the new total debt amount by multiplying the new total assets by the debt-equity ratio. Doing so, we find the new total debt is:

New total debt $=[$ Total debt $/($ Total debt + Total equity $)]($ New total assets $)$
New total debt $=[\$ 49,000 /(\$ 49,000+72,000)](\$ 151,513.04)$
New total debt $=\$ 61,356.52$

The additional borrowing is the difference between the new total debt and the current total debt, so:
Additional borrowing $=$ New total debt - Current total debt
Additional borrowing $=\$ 61,356.52-49,000$
Additional borrowing $=\$ 12,356.52$
The growth rate that could be achieved with no outside financing at all is the internal growth rate.
To find the internal growth rate we first need the return on assets, which is:
ROA $=$ Net income $/$ Total assets
ROA $=\$ 19,000 /(\$ 49,000+72,000)$
$\mathrm{ROA}=0.1570$ or $15.70 \%$
So, the internal growth rate is:
Internal growth rate $=[(\mathrm{ROA})(\mathrm{b})] /[1-(\mathrm{ROA})(\mathrm{b})]$
Internal growth rate $=[(0.1570)(0.7632)] /[1-(0.1570)(0.7632)]$
Internal growth rate $=.1362$ or $13.62 \%$
43. We can find the payout ratio from the sustainable growth rate formula. First, we need the return on equity. Using the Du Pont identity, we find the return on equity is:

ROE $=($ Profit margin $)($ Total asset turnover)(Equity multiplier)
ROE $=(0.06)(1.05)(1+0.40)$
ROE $=0.0882$ or $8.82 \%$
Now we can use the sustainable growth rate equation to find the retention ratio, which is:
Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
$0.13=[0.0882(\mathrm{~b})] /[1-0.0882(\mathrm{~b})]$
$\mathrm{b}=1.30$
So, the payout ratio is:
Payout ratio $=1-\mathrm{b}$
Payout ratio $=1-1.30$
Payout ratio $=-0.30$ or $-30 \%$
This is a negative dividend payout ratio of $130 \%$, which is impossible; the growth rate is not consistent with the other constraints. The lowest possible payout rate is zero, which corresponds to retention ratio of one, or total earnings retention. The maximum sustainable growth rate for this company is:

Sustainable growth rate $=[(\mathrm{ROE})(\mathrm{b})] /[1-(\mathrm{ROE})(\mathrm{b})]$
Sustainable growth rate $=[0.0882(1)] /[1-0.0882(1)]$
Sustainable growth rate $=0.0967$ or $9.67 \%$

## B-48 SOLUTIONS

44. Using the beginning of period total assets, the ROA is:

$$
\begin{aligned}
& \mathrm{ROA}_{\text {Begin }}=\$ 1,140 / \$ 10,294 \\
& \mathrm{ROA}_{\text {Begin }}=.1107 \text { or } 11.07 \%
\end{aligned}
$$

Using the end of period total assets, the ROA is:

$$
\begin{aligned}
& \mathrm{ROA}_{\text {End }}=\$ 1,140 / \$ 11,864 \\
& \mathrm{ROA}_{\text {End }}=.0961 \text { or } 9.61 \%
\end{aligned}
$$

The ROE using beginning of period equity is:

$$
\begin{aligned}
& \mathrm{ROE}_{\text {Begin }}=\$ 1,140 / \$ 4,449 \\
& \mathrm{ROE}_{\text {Begin }}=.2562 \text { or } 25.62 \%
\end{aligned}
$$

The ROE using the end of period equity is:

$$
\begin{aligned}
& \operatorname{ROE}_{\text {End }}=\$ 1,140 / \$ 5,257 \\
& \operatorname{ROE}_{\text {End }}=.2169 \text { or } 21.69 \%
\end{aligned}
$$

The retention ratio, which is one minus the dividend payout ratio, is:
$\mathrm{b}=1$ - Dividends/Net income
$\mathrm{b}=1-\$ 151 / \$ 1,140$
$\mathrm{b}=.8675$ or $86.75 \%$
With the growth rate equations, we need to use the ROA and ROE based on the end of period assets or equity, so the internal growth rate is:

```
Internal growth rate = [(ROA)(b)] / [1-(ROA)(b)]
Internal growth rate = [(.0961)(.8675)] / [1 - (.0961)(.8675)]
Internal growth rate =.0909 or 9.09%
```

And the sustainable growth rate is:

```
Sustainable growth rate = [(ROE)(b)]/ [1 - (ROE)(b)]
Sustainable growth rate = [(.2169)(.8675)]/ [1 - (.2169)(.8675)]
Sustainable growth rate = .2317 or 23.17%
```

Using ROA $\times \mathrm{b}$ and end of period assets to find the internal growth rate, we find:

```
Internal growth rate = ROA 
Internal growth rate =.0961 ×.8675
Internal growth rate = .0834 or 8.34%
```

And, using ROE $\times \mathrm{b}$ and the end of period equity to find the sustainable growth rate, we find:
Sustainable growth rate $=\mathrm{ROE}_{\text {End }} \times \mathrm{b}$
Sustainable growth rate $=.2169 \times .8675$
Sustainable growth rate $=.1881$ or $18.81 \%$
Using ROA $\times \mathrm{b}$ and beginning of period assets to find the internal growth rate, we find:
Internal growth rate $=\mathrm{ROA}_{\text {Begin }} \times \mathrm{b}$
Internal growth rate $=.1107 \times .8675$
Internal growth rate $=.0961$ or $9.61 \%$
And, using ROE $\times \mathrm{b}$ and the beginning of period equity to find the sustainable growth rate, we find:
Sustainable growth rate $=\mathrm{ROE}_{\text {Begin }} \times \mathrm{b}$
Sustainable growth rate $=.2562 \times .8675$
Sustainable growth rate $=.2223$ or $22.23 \%$
45. The expanded Du Pont table is shown on the next page. The ROE is $81.80 \%$.


# CHAPTER 4 <br> INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY 

## Answers to Concepts Review and Critical Thinking Questions

1. Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.
2. Future values grow (assuming a positive rate of return); present values shrink.
3. The future value rises (assuming a positive rate of return); the present value falls.
4. It depends. The large deposit will have a larger future value for some period, but after time, the smaller deposit with the larger interest rate will eventually become larger. The length of time for the smaller deposit to overtake the larger deposit depends on the amount deposited in each account and the interest rates.
5. It would appear to be both deceptive and unethical to run such an without a disclaimer or explanation.
6. It's a reflection of the time value of money. TMCC gets to use the $\$ 1,163$. If TMCC uses it wisely, it will be worth more than $\$ 10,000$ in thirty years.
7. This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a "call" feature. Such features are discussed at length in a later chapter.
8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $\$ 10,000$ ? Thus, our answer does depend on who is making the promise to repay.
9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.
10. The price would be higher because, as time passes, the price of the security will tend to rise toward $\$ 10,000$. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $\$ 10,000$ grows shorter, and the present value rises. In 2015 , the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC's financial position could deteriorate. Either event would tend to depress the security's price.

## B-52 SOLUTIONS

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The simple interest per year is:
$\$ 6,000 \times 0.08=\$ 480$
So, after 10 years, you will have:
$\$ 480 \times 10=\$ 4,800$ in interest.

The total balance will be $\$ 6,000+4,800=\$ 10,800$
With compound interest, we use the future value formula:
$\mathrm{FV}=\mathrm{PV}(1+r)^{\mathrm{t}}$
$\mathrm{FV}=\$ 6,000(1.08)^{10}=\$ 12,953.55$
The difference is:
$\$ 12,953.55-10,800=\$ 2,153.55$
2. To find the FV of a lump sum, we use:

$$
\begin{array}{ll}
\mathrm{FV}=\mathrm{PV}(1+r)^{t} & \\
& =\$ 7,206.44 \\
\mathrm{FV}=\$ 3,150(1.18)^{5} & =\$ 15,138.04 \\
\mathrm{FV}=\$ 8,453(1.06)^{10} & =\$ 526,461.25 \\
\mathrm{FV}=\$ 89,305(1.11)^{17} & =\$ 603,312.14
\end{array}
$$

3. To find the PV of a lump sum, we use:

$$
\begin{array}{ll}
\mathrm{PV}=\mathrm{FV} /(1+r)^{t} & \\
& \\
\mathrm{PV}=\$ 15,451 /(1.04)^{12} & =\$ 9,650.65 \\
\mathrm{PV}=\$ 51,557 /(1.09)^{4} & =\$ 36,524.28 \\
\mathrm{PV}=\$ 886,073 /(1.17)^{16} & =\$ 71,861.41 \\
\mathrm{PV}=\$ 901,450 /(1.20)^{21} & =\$ 19,594.56
\end{array}
$$

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$\mathrm{FV}=\$ 307=\$ 221(1+r)^{6}$
$r=(\$ 307 / \$ 221)^{1 / 6}-1$
$r=0.0563$ or $5.63 \%$
$\mathrm{FV}=\$ 905=\$ 425(1+r)^{7}$
$r=(\$ 905 / \$ 425)^{1 / 7}-1$
$r=0.1140$ or $11.40 \%$
$\mathrm{FV}=\$ 143,625=\$ 25,000(1+r)^{18}$
$r=(\$ 143,625 / \$ 25,000)^{1 / 18}-1$
$r=0.1020$ or $10.20 \%$
$\mathrm{FV}=\$ 255,810=\$ 40,200(1+r)^{21}$
$r=(\$ 255,810 / \$ 40,200)^{1 / 21}-1$
$r=0.0921$ or $9.21 \%$
5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $t$, we get:
$t=\ln (\mathrm{FV} / \mathrm{PV}) / \ln (1+r)$
$\mathrm{FV}=\$ 1,105=\$ 250(1.08)^{t}$
$t=\ln (\$ 1,105 / \$ 250) / \ln 1.08$
$t=19.31$ years
$\mathrm{FV}=\$ 3,700=\$ 1,941(1.05)^{t}$
$t=\ln (\$ 3,700 / \$ 1,941) / \ln 1.05$
$t=13.22$ years
$\mathrm{FV}=\$ 387,120=\$ 32,805(1.14)^{t}$
$t=\ln (\$ 387,120 / \$ 32,805) / \ln 1.14$
$t=18.84$ years

## B-54 SOLUTIONS

$\mathrm{FV}=\$ 198,212=\$ 32,500(1.24)^{t}$
$t=\ln (\$ 198,212 / \$ 32,500) / \ln 1.24$
$t=8.41$ years
6. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 290,000 / \$ 35,000)^{1 / 18}-1$
$r=0.1247$ or $12.47 \%$
7. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $t$, we get:
$t=\ln (\mathrm{FV} / \mathrm{PV}) / \ln (1+r)$
The length of time to double your money is:

```
FV = $2 = $1(1.09)
t= ln 2 / ln 1.09
t=8.04 years
```

The length of time to quadruple your money is:

```
FV = $4 = $1(1.09)
t= ln 4 / ln 1.09
t=16.09 years
```

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the slight difference in these answers is due to rounding). This is an important concept of time value of money.
8. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 1,300 / \$ 0.50)^{1 / 102}-1$
$r=0.0801$ or $8.01 \%$
9. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $t$, we get:
$t=\ln (\mathrm{FV} / \mathrm{PV}) / \ln (1+r)$
$\mathrm{FV}=\$ 160,000=\$ 30,000(1.047)^{t}$
$t=\ln (\$ 160,000 / \$ 30,000) / \ln 1.047$
$t=36.45$ years
10. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$\mathrm{PV}=\$ 800,000,000 /(1.08)^{20}$
$\mathrm{PV}=\$ 171,638,566$
11. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$\mathrm{PV}=\$ 2,000,000 /(1.11)^{80}$
$\mathrm{PV}=\$ 473.36$
12. To find the FV of a lump sum, we use:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
$\mathrm{FV}=\$ 50(1.057)^{108}$
$\mathrm{FV}=\$ 19,909.88$

## B-56 SOLUTIONS

13. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
```
\(r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1\)
\(r=(\$ 1,225,000 / \$ 150)^{1 / 111}-1\)
\(r=0.0845\) or \(8.45 \%\)
```

To find what the check will be in 2040, we use the FV of a lump sum, so:

```
\(\mathrm{FV}=\mathrm{PV}(1+r)^{t}\)
\(\mathrm{FV}=\$ 1,225,000(1.0845)^{34}\)
FV \(=\$ 19,338,380.03\)
```

14. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 9,000 / \$ 3)^{1 / 127}-1$
$r=.0651$ or $6.51 \%$
15. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 10,311,500 / \$ 12,377,500)^{1 / 4}-1$
$r=-.0446$ or $-4.46 \%$

## Intermediate

16. $a$. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 10,000 / \$ 1,163)^{1 / 30}-1$
$r=0.0744$ or $7.44 \%$
b. Using the FV formula and solving for the interest rate, we get:
```
\(r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1\)
\(r=(\$ 2,500 / \$ 1,163)^{1 / 9}-1\)
\(r=0.0888\) or \(8.88 \%\)
```

c. Using the FV formula and solving for the interest rate, we get:

```
\(r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1\)
\(r=(\$ 10,000 / \$ 2,500)^{1 / 21}-1\)
\(r=0.0682\) or \(6.82 \%\)
```

17. To find the PV of a lump sum, we use:
```
PV = FV / (1+r)t
PV = $160,000 / (1.1075) }\mp@subsup{}{}{10
PV = $57,634.51
```

18. To find the FV of a lump sum, we use:
```
\(\mathrm{FV}=\mathrm{PV}(1+r)^{t}\)
\(\mathrm{FV}=\$ 5,000(1.11)^{45}\)
\(\mathrm{FV}=\$ 547,651.21\)
```

If you wait 10 years, the value of your deposit at your retirement will be:
$\mathrm{FV}=\$ 5,000(1.11)^{35}$
$\mathrm{FV}=\$ 192,874.26$
Better start early!

## B-58 SOLUTIONS

19. Even though we need to calculate the value in eight years, we will only have the money for six years, so we need to use six years as the number of periods. To find the FV of a lump sum, we use:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
$\mathrm{FV}=\$ 15,000(1.08)^{6}$
$\mathrm{FV}=\$ 23,803.11$
20. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
```
FV = PV (1 +r)
$160,000 = $30,000(1.09)}\mp@subsup{}{}{t
t= ln($160,000 / $30,000) / ln 1.09
t=19.42 years
```

From now, you'll wait $2+19.42=21.42$ years
21. To find the FV of a lump sum, we use:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
In Regency Bank, you will have:
$\mathrm{FV}=\$ 7,000(1.01)^{240}$
$\mathrm{FV}=\$ 76,247.88$
And in King Bank, you will have:
$\mathrm{FV}=\$ 7,000(1.12)^{20}$
$\mathrm{FV}=\$ 67,524.05$
22. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. We also need to be careful about the number of periods. Since the length of the compounding is three months and we have 24 months, there are eight compounding periods. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
Solving for $r$, we get:
$r=(\mathrm{FV} / \mathrm{PV})^{1 / t}-1$
$r=(\$ 3 / \$ 1)^{1 / 8}-1$
$r=0.1472$ or $14.72 \%$
23. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
$\$ 3,600=\$ 1,500(1.0045)^{t}$
$t=\ln (\$ 3,600 / \$ 1,500) / \ln 1.0045$
$t=194.99$ months
24. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$P V=\$ 75,000 /(1.0055)^{120}$
$\mathrm{PV}=\$ 38,834.01$
25. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
So, if you can earn 11 percent, you will need to invest:
$\mathrm{PV}=\$ 1,000,000 /(1.12)^{45}$
$\mathrm{PV}=\$ 6,098.02$
And if you can earn 5 percent, you will need to invest:
$P V=\$ 1,000,000 /(1.06)^{45}$
$\mathrm{PV}=\$ 72,650.07$

## Challenge

26. In this case, we have an investment that earns two different interest rates. We will calculate the value of the investment at the end of the first 20 years then use this value with the second interest rate to find the final value at the end of 40 years. Using the future value equation, at the end of the first 20 years, the account will be worth:

Value in 20 years $=\mathrm{PV}(1+r)^{t}$
Value in 20 years $=\$ 10,000(1.08)^{20}$
Value in 20 years $=\$ 46,609.57$
Now we can find out how much this will be worth 20 years later at the end of the investment. Using the future value equation, we find:

Value in 40 years $=\mathrm{PV}(1+r)^{t}$
Value in 40 years $=\$ 46,609.57(1.12)^{20}$
Value in 40 years $=\$ 449,609.59$

It is irrelevant which interest rate is offered when as long as each interest rate is offered for 20 years. We can find the value of the initial investment in 40 years with the following:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}\left(1+r_{1}\right)^{t}\left(1+r_{2}\right)^{t} \\
& \mathrm{FV}=\$ 10,000(1.08)^{20}(1.12)^{20} \\
& \mathrm{FV}=\$ 449,609.59
\end{aligned}
$$

With the commutative property of multiplication, it does not matter which order the interest rates occur, the final value will always be the same.

## Calculator Solutions

1. 



$$
\$ 6,953.55-10(\$ 480)=\$ 2,153.55
$$

2. 

| Enter | 5 | $18 \%$ | $\pm \$ 3,150$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  |  | $\$ 7,206.44$ |


| Enter | 10 | 6\% | $\pm$ \$8,453 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  |  | \$15,138.04 |
| Enter | 17 | 11\% | $\pm$ \$89,305 |  |  |
|  | N | I/Y | PV | PMT | FV |


| Enter | 20 | $5 \%$ | $\pm \$ 227,382$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  | $\$ 603,312.14$ |  |

3. 



| Enter | 4 | 9\% |  |  | \$51,557 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | 36,52 |  |  |



4.

| Enter | 6 |  | $\pm \$ 221$ |  | \$307 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 5.63\% |  |  |  |


| Enter | 7 |  | $\pm \$ 425$ |  | \$905 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 1.40\% |  |  |  |


| Enter | 18 |  |  | $\pm \$ 25,000$ | P143,625 |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  | $10.20 \%$ |  |  |  |


| Enter | 21 |  | $\pm$ \$40,200 |  | \$255,810 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 9.21\% |  |  |  |

5. 

| Enter |  | 8\% | $\pm \$ 250$ |  | \$1,105 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 19.31 |  |  |  |  |
| Enter |  | 5\% | $\pm$ 1,941 |  | \$3,700 |
|  | N | I/Y | PV | PMT | FV |


| Enter | N | $\begin{aligned} & 14 \% \\ & \mathbf{I} / \mathbf{Y} \end{aligned}$ | $\frac{ \pm \$ 32,805}{\mathrm{PV}}$ | PMT | $\begin{gathered} \$ 387,120 \\ \text { FV } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Solve for | 18.84 |  |  |  |  |
| Enter |  | 24\% | $\pm$ \$32,500 |  | \$198,212 |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 8.81 |  |  |  |  |

6. 

| Enter | 18 |  | $\pm$ 35,000 |  | \$290,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | $12.47 \%$ |  |  |  |

7. 

| Enter |  | 9\% | $\pm$ \$1 |  | \$2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 8.04 |  |  |  |  |


| Enter |  | 9\% | $\pm \$ 1$ |  | \$4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 16.09 |  |  |  |  |

8. 


9.

| Enter |  | 4.7\% | $\pm$ 30,000 |  | \$160,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 36.45 |  |  |  |  |

10. 


11.

12.

13.

| Enter | 111 |  | $\pm \$ 150$ |  | \$1,225,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 8.45\% |  |  |  |


| Enter | 34 | $8.45 \%$ | $\pm \$ 1,225,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  |  | $\$ 19,338,380$ |

14. 

| Enter | 127 |  | $\pm \$ 3$ |  | $\$ 9,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | PV | PMT | FV |
| Solve for |  | $6.51 \%$ |  |  |  |

15. 


16. $a$.

| Enter | 30 |  | $\pm$ \$1,163 |  | \$10,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 7.44\% |  |  |  |
| $b$. |  |  |  |  |  |
| Enter | 9 |  | $\pm \$ 1,163$ |  | \$2,500 |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 8.88\% |  |  |  |
| c. |  |  |  |  |  |
| Enter | 21 |  | $\pm$ 2,500 |  | \$10,000 |
|  | N | //Y | PV | PMT | FV |
| Solve for |  | 6.82\% |  |  |  |

17. 

| Enter | 10 | $10.75 \%$ |  |  | $\$ 160,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | PV | PMT | FV |
| Solve for |  |  | $-\$ 57,634.51$ |  |  |

18. 



| Enter | 35 | $11 \%$ | $\pm \$ 5,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | PV | PMT | FV |
| Solve for |  |  |  |  | $\$ 192,874.26$ |

19. 

| Enter | 6 | 8\% | $\pm$ \$15,000 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  |  | \$23,803.11 |
| 20. |  |  |  |  |  |
| Enter |  | 9\% | $\pm$ \$30,000 |  | \$160,000 |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 19.42 |  |  |  |  |

You must wait $2+19.42=21.42$ years.
21.

| Enter | 240 | 1\% | $\pm \$ 7,000$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  |  | \$76,247.88 |
| Enter | 20 | 12\% | $\pm$ 7,000 |  |  |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  |  | \$67,524.05 |

22. 

| Enter | 8 |  | $\pm$ \$1 |  | \$3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |

23. 

| Enter |  | 0.45\% | $\pm \$ 1,500$ |  | \$3,600 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for | 94.99 |  |  |  |  |

24. 


25.

Enter
Solve for

| 45 | $12 \%$ |
| :--- | :--- |
| $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ |


| PV |  |
| :---: | :---: | :---: |
| $-\$ 6,098.02$ | PMT$\$ 1,000,000$ <br> FV |


| Enter | 45 | 6\% |  |  | \$1,000,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | P | PMT | FV |
| e |  |  | 2,6 |  |  |

26. 

Enter
Solve for

$\pm \begin{aligned} & \text { PV } \\ & \text { PV }\end{aligned}$

```
PMT
```

FV
\$46,609.57

Enter
Solve for
I/Y


PMT


Enter


PMT
\$12,000
FV

## CHAPTER 5

## DISCOUNTED CASH FLOW VALUATION

## Answers to Concepts Review and Critical Thinking Questions

1. Assuming positive cash flows and a positive interest rate, both the present and the future value will rise.
2. Assuming positive cash flows and a positive interest rate, the present value will fall, and the future value will rise.
3. It's deceptive, but very common. The deception is particularly irritating given that such lotteries are usually government sponsored!
4. The most important consideration is the interest rate the lottery uses to calculate the lump sum option. If you can earn an interest rate that is higher than you are being offered, you can create larger annuity payments. Of course, taxes are also a consideration, as well as how badly you really need $\$ 5$ million today.
5. If the total money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.
6. The better deal is the one with equal installments.
7. Yes, they should. APRs generally don't provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.
8. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.
9. The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.
10. The problem is that the subsidy makes it easier to repay the loan, not obtain it. However, the ability to repay the loan depends on future employment, not current need. For example, consider a student who is currently needy, but is preparing for a career in a high-paying area (such as corporate finance!). Should this student receive the subsidy? How about a student who is currently not needy, but is preparing for a relatively low-paying job (such as becoming a college professor)?

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
PV@ $10 \%=\$ 800 / 1.10+\$ 500 / 1.10^{2}+\$ 1,300 / 1.10^{3}+\$ 1,480 / 1.10^{4}=\$ 3,128.07$
PV@ $18 \%=\$ 800 / 1.18+\$ 500 / 1.18^{2}+\$ 1,300 / 1.18^{3}+\$ 1,480 / 1.18^{4}=\$ 2,591.65$
PV@24\% = \$800 / 1.24 + \$500/1.24 ${ }^{2}+\$ 1,300 / 1.24^{3}+\$ 1,480 / 1.24^{4}=\$ 2,278.18$
2. To find the PVA, we use the equation:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
At a 6 percent interest rate:
$\mathrm{X} @ 5 \%: \operatorname{PVA}=\$ 5,500\left\{\left[1-(1 / 1.06)^{9}\right] / .06\right\}=\$ 37,409.31$
$\mathrm{Y} @ 5 \%: \quad \mathrm{PVA}=\$ 8,000\left\{\left[1-(1 / 1.06)^{5}\right] / .06\right\}=\$ 33,698.91$
And at a 22 percent interest rate:
$\mathrm{X} @ 22 \%: \mathrm{PVA}=\$ 5,500\left\{\left[1-(1 / 1.22)^{9}\right] / .22\right\}=\$ 20,824.57$
$\mathrm{Y} @ 22 \%: \mathrm{PVA}=\$ 8,000\left\{\left[1-(1 / 1.22)^{5}\right] / .22\right\}=\$ 22,909.12$
Notice that the PV of Investment X has a greater PV at a 6 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger annual payments. At a higher interest rate, getting these payments early are more important since the cost of waiting (the interest rate) is so much greater.

## B-68 SOLUTIONS

3. To solve this problem, we must find the FV of each cash flow and sum. To find the FV of a lump sum, we use:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
FV@ $8 \%=\$ 700(1.08)^{3}+\$ 900(1.08)^{2}+\$ 1,400(1.08)+\$ 2,000=\$ 5,443.56$
FV@ $11 \%=\$ 700(1.11)^{3}+\$ 900(1.11)^{2}+\$ 1,400(1.11)+\$ 2,000=\$ 5,620.23$
FV@ $24 \%=\$ 700(1.24)^{3}+\$ 900(1.24)^{2}+\$ 1,400(1.24)+\$ 2,000=\$ 6,454.48$
Notice, since we are finding the value at Year 4, the cash flow at Year 4 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.
4. To find the PVA, we use the equation:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA@ 15 yrs: $\quad \operatorname{PVA}=\$ 7,000\left\{\left[1-(1 / 1.09)^{15}\right] / .09\right\}=\$ 56,424.82$
PVA@40 yrs: $\quad$ PVA $=\$ 7,000\left\{\left[1-(1 / 1.09)^{40}\right] / .09\right\}=\$ 75,301.52$
PVA@75 yrs: $\quad \mathrm{PVA}=\$ 7,000\left\{\left[1-(1 / 1.09)^{75}\right] / .09\right\}=\$ 77,656.48$
To find the PV of a perpetuity, we use the equation:

$$
\begin{aligned}
& \mathrm{PV}=C / r \\
& \mathrm{PV}=\$ 7,000 / .09 \\
& \mathrm{PV}=\$ 77,777.78
\end{aligned}
$$

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75 -year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $\$ 121.30$.
5. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

$$
\begin{aligned}
& \text { PVA }=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right) \\
& \text { PVA }=\$ 20,000=\$ C\left\{\left[1-(1 / 1.085)^{12}\right] / .085\right\}
\end{aligned}
$$

We can now solve this equation for the annuity payment. Doing so, we get:
$C=\$ 20,000 / 9.46334$
$C=\$ 2,723.06$
6. To find the PVA, we use the equation:

PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 50,000\left\{\left[1-(1 / 1.0825)^{9}\right] / .0825\right\}$
PVA $=\$ 309,123.20$
The present value of the revenue is greater than the cost, so your company can afford the equipment.
7. Here we need to find the FVA. The equation to find the FVA is:
$\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}$
FVA for 20 years $=\$ 3,000\left[\left(1.095^{20}-1\right) / .095\right]$
FVA for 20 years $=\$ 162,366.70$
FVA for 40 years $=\$ 3,000\left[\left(1.095^{40}-1\right) / .095\right]$
FVA for 40 years $=\$ 1,159,559.98$
Notice that doubling the number of periods more than doubles the FVA.
8. Here we have the FVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the FVA equation:
$\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}$
$\$ 30,000=\$ C\left[\left(1.0525^{8}-1\right) / .0525\right]$
We can now solve this equation for the annuity payment. Doing so, we get:
$C=\$ 30,000 / 9.63492$
$C=\$ 3,113.68$
9. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
$\$ 50,000=C\left\{\left[1-(1 / 1.08)^{7}\right] / .08\right\}$
We can now solve this equation for the annuity payment. Doing so, we get:
$C=\$ 50,000 / 5.20637$
$C=\$ 9,603.62$

## B-70 SOLUTIONS

10. This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

$$
\begin{aligned}
& \mathrm{PV}=C / r \\
& \mathrm{PV}=\$ 25,000 / .07=\$ 357,142.86
\end{aligned}
$$

11. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$$
\begin{aligned}
& \mathrm{PV}=C / r \\
& \$ 400,000=\$ 25,000 / r
\end{aligned}
$$

We can now solve for the interest rate as follows:

$$
\begin{aligned}
& r=\$ 25,000 / \$ 400,000 \\
& r=.0625 \text { or } 6.25 \%
\end{aligned}
$$

12. For discrete compounding, to find the EAR, we use the equation:
$\operatorname{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
$\operatorname{EAR}=[1+(.09 / 4)]^{4}-1=.0931$ or $9.31 \%$
$\operatorname{EAR}=[1+(.13 / 12)]^{12}-1=.1380$ or $13.80 \%$
$\operatorname{EAR}=[1+(.16 / 365)]^{365}-1=.1735$ or $17.35 \%$
$\operatorname{EAR}=[1+(.19 / 2)]^{2}-1=.1990$ or $19.90 \%$
13. Here we are given the EAR and need to find the APR. Using the equation for discrete compounding:
$\mathrm{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
We can now solve for the APR. Doing so, we get:

$$
\begin{aligned}
& \mathrm{APR}=m\left[(1+\mathrm{EAR})^{1 / m}-1\right] \\
& \mathrm{EAR}=.10=[1+(\mathrm{APR} / 2)]^{2}-1 \\
& \mathrm{EAR}=.14=[1+(\mathrm{APR} / 12)]^{12}-1 \\
& \mathrm{EAR}=.09=[1+(\mathrm{APR} / 52)]^{52}-1 \\
& \mathrm{EAR}=.16=[1+(\mathrm{APR} / 365)]^{365}-1
\end{aligned}
$$

$$
\begin{array}{ll}
\mathrm{APR}=2\left[(1.10)^{1 / 2}-1\right] & =9.76 \% \\
\mathrm{APR}=12\left[(1.14)^{1 / 12}-1\right] & =13.17 \% \\
\mathrm{APR}=52\left[(1.09)^{1 / 52}-1\right] & =8.62 \% \\
\mathrm{APR}=365\left[(1.16)^{1 / 365}-1\right] & =14.85 \%
\end{array}
$$

14. For discrete compounding, to find the EAR, we use the equation:
$\operatorname{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
So, for each bank, the EAR is:
First National: $\mathrm{EAR}=[1+(.124 / 12)]^{12}-1=.1313$ or $13.13 \%$
First United: $\quad \mathrm{EAR}=[1+(.127 / 2)]^{2}-1=.1310$ or $13.10 \%$
For a borrower, First United would be preferred since the EAR of the loan is lower. Notice that the higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.
15. The reported rate is the APR, so we need to convert the EAR to an APR as follows:
$\operatorname{EAR}=[1+(\mathrm{APR} / m)]^{m}-1$
$\mathrm{APR}=m\left[(1+\mathrm{EAR})^{1 / m}-1\right]$
$\mathrm{APR}=365\left[(1.18)^{1 / 365}-1\right]=.1656$ or $16.56 \%$
This is deceptive because the borrower is actually paying annualized interest of $18 \%$ per year, not the $16.56 \%$ reported on the loan contract.
16. For this problem, we simply need to find the FV of a lump sum using the equation:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
It is important to note that compounding occurs semiannually. To account for this, we will divide the interest rate by two (the number of compounding periods in a year), and multiply the number of periods by two. Doing so, we get:
$\mathrm{FV}=\$ 1,280[1+(.11 / 2)]^{26}$
$\mathrm{FV}=\$ 5,149.61$
17. For this problem, we simply need to find the FV of a lump sum using the equation:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365 . Doing so, we get:

FV in 5 years $=\$ 5,000[1+(.045 / 365)]^{5(365)}=\$ 6,261.53$
FV in 10 years $=\$ 5,000[1+(.045 / 365)]^{10(365)}=\$ 7,841.34$
FV in 20 years $=\$ 5,000[1+(.045 / 365)]^{20(365)}=\$ 12,297.33$

## B-72 SOLUTIONS

18. For this problem, we simply need to find the PV of a lump sum using the equation:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
It is important to note that compounding occurs on a daily basis. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365 . Doing so, we get:
$\mathrm{PV}=\$ 75,000 /\left[(1+.12 / 365)^{6(365)}\right]$
$\mathrm{PV}=\$ 36,510.74$
19. The APR is simply the interest rate per period times the number of periods in a year. In this case, the interest rate is 20 percent per month, and there are 12 months in a year, so we get:

$$
\begin{aligned}
& \mathrm{APR}=12(20 \%) \\
& \mathrm{APR}=240 \%
\end{aligned}
$$

To find the EAR, we use the EAR formula:

$$
\begin{aligned}
& \mathrm{EAR}=[1+(\mathrm{APR} / m)]^{m}-1 \\
& \mathrm{EAR}=(1+.20)^{12}-1 \\
& \mathrm{EAR}=7.9161 \text { or } 791.61 \%
\end{aligned}
$$

Notice that we didn't need to divide the APR by the number of compounding periods per year. We do this division to get the interest rate per period, but in this problem we are already given the interest rate per period.
20. We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
\(\$ 69,500=\$ C\left[1-\left\{1 /[1+(.086 / 12)]^{60}\right\} /(.086 / 12)\right]\)
```

Solving for the payment, we get:
C $=\$ 69,500 / 48.62687$
$C=\$ 1,429.25$
To find the EAR, we use the EAR equation:

```
EAR \(=[1+(\mathrm{APR} / m)]^{m}-1\)
EAR \(=[1+(.086 / 12)]^{12}-1\)
\(\mathrm{EAR}=.0895\) or \(8.95 \%\)
```

21. Here we need to find the length of an annuity. We know the interest rate, the $P V$, and the payments. Using the PVA equation:

$$
\begin{aligned}
& \text { PVA }=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right) \\
& \$ 13,850=\$ 500\left\{\left[1-(1 / 1.011)^{t}\right] / .011\right\}
\end{aligned}
$$

Now we solve for $t$ :
$1 / 1.011^{t}=1-[(\$ 13,850)(.011) /(\$ 500)]$
$1.011^{t}=1 /(0.6953)=1.4382$
$t=\ln 1.4382 / \ln 1.011$
$t=33.22$ months
22. Here we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}(1+r) \\
& \$ 5=\$ 4(1+r) \\
& r=\$ 5 / \$ 4-1 \\
& r=.2500 \text { or } 25.00 \% \text { per week }
\end{aligned}
$$

The interest rate is $25.00 \%$ per week. To find the APR, we multiply this rate by the number of weeks in a year, so:
$\mathrm{APR}=(52) 25.00 \%=1,300.00 \%$
And using the equation to find the EAR, we find:
EAR $=[1+(\mathrm{APR} / m)]^{m}-1$
EAR $=[1+.2500]^{52}-1$
$\operatorname{EAR}=109,475.4425$ or $10,947,544.25 \%$
23. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$$
\begin{aligned}
& \mathrm{PV}=C / r \\
& \$ 160,000=\$ 2,500 / r
\end{aligned}
$$

We can now solve for the interest rate as follows:
$r=\$ 2,500 / \$ 160,000$
$r=.0156$ or $1.56 \%$ per month
The interest rate is $1.56 \%$ per month. To find the APR, we multiply this rate by the number of months in a year, so:

$$
\begin{aligned}
& \mathrm{APR}=(12) 1.56 \% \\
& \mathrm{APR}=18.75 \%
\end{aligned}
$$

## B-74 SOLUTIONS

And using the equation to find the EAR, we find:
EAR $=[1+(\text { APR } / m)]^{m}-1$
EAR $=[1+.0156]^{12}-1$
$\mathrm{EAR}=.2045$ or $20.45 \%$
24. This problem requires us to find the FVA. The equation to find the FVA is:

```
FVA = C{[(1+r)t - 1]/r}
FVA = $300[{[1 + (.12/12)] ]60 - 1} / (.12/12)]
FVA = $1,048,489.24
```

25. In the previous problem, the cash flows are monthly and the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate times with the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

EAR $=[1+(\operatorname{APR} / m)]^{m}-1$
$\operatorname{EAR}=[1+(.12 / 12)]^{12}-1$
$\mathrm{EAR}=.1268$ or $12.68 \%$
Using the FVA equation, we get:
$\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}$
FVA $=\$ 3,600\left[\left(1.1268^{30}-1\right) / .1157\right]$
FVA $=\$ 992,065.28$
26. The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
PVA \(=\$ 2,200\left\{\left[1-(1 / 1.009)^{16}\right] / .009\right\}\)
PVA \(=\$ 32,646.61\)
```

27. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

$$
\begin{aligned}
& \mathrm{PV}=\mathrm{FV} /(1+r)^{t} \\
& \mathrm{PV}=\$ 300 / 1.095+\$ 900 / 1.095^{2}+\$ 700 / 1.095^{3}+\$ 600 / 1.095^{4} \\
& \mathrm{PV}=\$ 1,975.08
\end{aligned}
$$

28. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$\mathrm{PV}=\$ 2,400 / 1.0816+\$ 3,200 / 1.0816^{2}+\$ 6,800 / 1.0816^{3}+\$ 8,100 / 1.0816^{4}$
$\mathrm{PV}=\$ 16,247.04$

## Intermediate

29. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:
$.07(10)=.7$
First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of \$1, or:
$(1+r)^{10}$
Setting the two equal, we get:
$(.07)(10)=(1+r)^{10}-1$
$r=1.7^{1 / 10}-1$
$r=.0545$ or $5.45 \%$
30. We need to use the PVA due equation, which is:

PVA $_{\text {due }}=(1+r)$ PVA
Using this equation:
$\mathrm{PVA}_{\text {due }}=\$ 48,000=[1+(.0745 / 12)] \times C\left[\left\{1-1 /[1+(.0745 / 12)]^{60}\right\} /(.0745 / 12)\right.$
$\$ 47,703.84=\$ C\left\{1-\left[1 /(1+.0745 / 12)^{60}\right]\right\} /(.0745 / 12)$
$C=\$ 954.75$
Notice, when we find the payment for the PVA due, we simply discount the PV of the annuity due back one period. We then use this value as the PV of an ordinary annuity.
31. Here we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:
$\mathrm{FV}=\$ 10,000[1+(.015 / 12)]^{6}$
$\mathrm{FV}=\$ 10,075.23$

## B-76 SOLUTIONS

This is the balance in six months. The FV in another six months will be:

$$
\begin{aligned}
& \mathrm{FV}=\$ 10,075.23[1+(.18 / 12)]^{6} \\
& \mathrm{FV}=\$ 11,016.70
\end{aligned}
$$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the principal. The interest accrued is:

```
Interest = $11,016.70-10,000.00
Interest = $1,016.70
```

32. We will calculate the time we must wait if we deposit in the bank that pays simple interest. The interest amount we will receive each year in this bank will be:
```
Interest = $89,000(.05)
Interest = $4,450 per year
```

The deposit will have to increase by the difference between the amount we need by the amount we originally deposit with divided by the interest earned per year, so the number of years it will take in the bank that pays simple interest is:

Years to wait $=(\$ 175,000-89,000) / \$ 4,450$
Years to wait $=19.33$ years
To find the number of years it will take in the bank that pays compound interest, we can use the future value equation for a lump sum and solve for the periods. Doing so, we find:

```
\(\mathrm{FV}=\mathrm{PV}(1+r)^{t}\)
\(\$ 175,000=\$ 89,000[1+(.05 / 12)]^{t}\)
\(t=162.61\) months or 13.55 years
```

33. Here we need to find the future value of a lump sum. We need to make sure to use the correct number of periods. So, the future value after one year will be:
```
FV = PV (1 +r)t
FV = $1(1.0105) 
FV = $1.13
```

And the future value after two years will be:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
$\mathrm{FV}=\$ 1(1.0105)^{24}$
$\mathrm{FV}=\$ 1.28$
34. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Even though the currency is pounds and not dollars, we can still use the same time value equations. Using the PVA equation:

PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
$£ 440=£ 60\left[\left\{1-[1 /(1+r)]^{31}\right\} / r\right]$
To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:
$r=13.36 \%$
Not bad for an English Literature major!
35. Here we need to compare two cash flows. The only way to compare cash flows is to find the value of the cash flows at a common time, so we will find the present value of each cash flow stream. Since the cash flows are monthly, we need to use the monthly interest rate, which is:

Monthly rate $=.07 / 12$
Monthly rate $=.0058$ or $.58 \%$
The value today of the $\$ 6,200$ monthly salary is:
PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 7,500\left\{\left[1-(1 / 1.0058)^{24}\right] / .0058\right\}$
PVA $=\$ 167,513.24$
To find the value of the second option, we find the present value of the monthly payments and add the bonus. We can add the bonus since it is paid today. So:

PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 6,000\left\{\left[1-(1 / 1.0058)^{24}\right] / .0058\right\}$
PVA $=\$ 134,010.60$
So, the total value of the second option is:
Value of second option $=\$ 134,010.60+30,000$
Value of second option $=\$ 164,010.60$
The difference in the value of the two options today is:
Difference in value today $=\$ 167,513.24-164,010.60$
Difference in value today $=\$ 3,502.65$

## B-78 SOLUTIONS

What if we found the future value of the two cash flows? For the annual salary, the future value will be:

```
\(\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}\)
FVA \(=\$ 7,500\left\{\left[(1+.0058)^{24}-1\right] / .0058\right\}\)
\(\mathrm{FVA}=\$ 192,607.74\)
```

To find the future value of the second option we also need to find the future value of the bonus as well. So, the future value of this option is:

$$
\begin{aligned}
& \mathrm{FV}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}+\mathrm{PV}(1+r)^{t} \\
& \mathrm{FV}=\$ 6,000\left\{\left[(1+.0058)^{24}-1\right] / .0058\right\}+\$ 30,000(1+.0058)^{24} \\
& \mathrm{FV}=\$ 188,580.37
\end{aligned}
$$

So, the first option is still the better choice. The difference between the two options now is:
Difference in future value $=\$ 192,607.74-188,580.37$
Difference in future value $=\$ 4,027.37$
No matter when you compare two cash flows, the cash flow with the greatest value on one period will always have the greatest value in any other period. Here's a question for you: What is the future value of $\$ 3,502.65$ (the difference in the cash flows at time zero) in 24 months at an interest rate of .58 percent per month? With no calculations, you know the future value must be $\$ 4,027.37$, the difference in the cash flows at the same time!
36. The cash flows are an annuity, so we can use the present value of an annuity equation. Doing so, we find:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
PVA \(=\$ 15,000\left[1-(1 / 1.13)^{20} / .13\right]\)
PVA \(=\$ 105,371.27\)
```

37. The investment we should choose is the investment with the higher rate of return. We will use the future value equation to find the interest rate for each option. Doing so, we find the return for Investment G is:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}(1+r)^{t} \\
& \$ 120,000=\$ 75,000\left(1+r+{ }^{6}\right. \\
& r=(\$ 120,000 / \$ 75,000)^{1 / 6}-1 \\
& r=.0815 \text { or } 8.15 \% \\
& \text { And, the return for Investment } \mathrm{H} \text { is: }
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}(1+r)^{t} \\
& \$ 220,000=\$ 75,000(1+r)^{13} \\
& r=(\$ 220,000 / \$ 75,000)^{1 / 13}-1 \\
& r=.0863 \text { or } 8.63 \%
\end{aligned}
$$

So, we should choose Investment H .
38. The present value of an annuity falls as $r$ increases, and the present value of an annuity rises as $r$ decreases. The future value of an annuity rises as $r$ increases, and the future value of an annuity falls as $r$ decreases.

Here we need to calculate the present value of an annuity for different interest rates. Using the present value of an annuity equation and an interest rate of 10 percent, we get:

```
PVA =C({1-[1/(1+r)] } } r)
PVA = $10,000{[1-(1/1.10) }\mp@subsup{}{}{10}]/.10
PVA = $61,445.67
```

At an interest rate of 5 percent, the present value of the annuity is:

```
PVA \(=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
PVA \(=\$ 10,000\left\{\left[1-(1 / 1.05)^{10}\right] / .05\right\}\)
PVA \(=\$ 77,217.35\)
```

And, at an interest rate of 15 percent, the present value of the annuity is:
PVA $=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 10,000\left\{\left[1-(1 / 1.15)^{10}\right] / .15\right\}$
PVA $=\$ 50,187.69$
39. Here we are given the future value of an annuity, the interest rate, and the number of payments. We need to find the number of periods of the annuity payments. So, we can solve the future value of an annuity equation for the number of periods as follows:

```
\(\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}\)
\(\$ 50,000=\$ 250\left[\left\{[1+(.10 / 12)]^{t}-1\right\} /(.10 / 12)\right]\)
\(200=\left\{[1+(.10 / 12)]^{t}-1\right\} /(.10 / 12)\)
\(1.667=(1+.0083)^{t}-1\)
\(2.667=(1.0083)^{t}\)
\(\ln 2.667=t \ln 1.0083\)
\(t=\ln 2.667 / \ln 1.0083\)
\(t=118.19\) payments
```

40. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:
```
PVA \(=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
\(\$ 75,000=\$ 1,600\left[\left\{1-[1 /(1+r)]^{60}\right\} / r\right]\)
```


## B-80 SOLUTIONS

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:
$r=.00848$ or $.848 \%$

This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12 , so the APR is:

```
APR = .00848 × 12
APR = .1018 or 10.18%
```

41. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:
```
\(\mathrm{PV}=\mathrm{FV} /(1+r)^{t}\)
\(\mathrm{PV}=\$ 24,000,000+\$ 24,000,000 / 1.11+\$ 24,000,000 / 1.11^{2}+\$ 24,000,000 / 1.11^{3}+\$ 27,000,000 / 1.11^{4}\)
    \(+\$ 25,000,000 / 1.11^{5}+\$ 26,000,000 / 1.11^{6}+\$ 26,000,000 / 1.11^{7}+\$ 26,000,000 / 1.11^{8}\)
    \(+\$ 26,000,000 / 1.11^{9}\)
\(\mathrm{PV}=\$ 163,141,086.06\)
```

42. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

$$
\begin{aligned}
\mathrm{PV}= & \mathrm{FV} /(1+r)^{t} \\
\mathrm{PV}= & \$ 16,000,000+\$ 18,500,000 / 1.11+\$ 21,000,000 / 1.11^{2}+\$ 23,500,000 / 1.11^{3}+\$ 20,000,000 / 1.11^{4} \\
& \quad+\$ 20,000,000 / 1.11^{5}+\$ 18,000,000 / 1.11^{6}+\$ 20,000,000 / 1.11^{7} \\
\mathrm{PV}= & \$ 113,170,277.46
\end{aligned}
$$

43. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. First, we need to find the amount borrowed since it is only 80 percent of the building value. So, the amount borrowed is:

Amount borrowed $=.80(\$ 2,500,000)$
Amount borrowed $=\$ 2,000,000$

Now we can use the PVA equation:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
$\$ 2,000,000=\$ 13,400\left[\left\{1-[1 /(1+r)]^{360}\right\} / r\right]$

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:
$r=.00589$ or $.589 \%$
This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12 , so the APR is:
$\mathrm{APR}=.00589 \times 12$
$\mathrm{APR}=.0707$ or $7.07 \%$
And the EAR is:
EAR $=[1+(\text { APR } / m)]^{m}-1$
EAR $=[1+.00589]^{12}-1$
$\mathrm{EAR}=.0730$ or $7.30 \%$
44. Here, we have two cash flow streams that will be combined in the future. To find the withdrawal amount, we need to know the present value, as well as the interest rate and periods, which are given. The present value of the retirement account is the future value of the stock and bond account. We need to find the future value of each account and add the future values together. For the bond account the future value is the value of the current savings plus the value of the annual deposits. So, the future value of the bond account will be:

$$
\begin{aligned}
& \mathrm{FV}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}+\mathrm{PV}(1+r)^{t} \\
& \mathrm{FV}=\$ 9,000\left\{\left[(1+.075)^{10}-1\right] / .075\right\}+\$ 150,000(1+.075)^{10} \\
& \mathrm{FV}=\$ 436,478.52
\end{aligned}
$$

The total value of the stock account at retirement will be the future value of a lump sum, so:
$\mathrm{FV}=\mathrm{PV}(1+r)^{t}$
$\mathrm{FV}=\$ 450,000(1+.115)^{10}$
$\mathrm{FV}=\$ 1,336,476.07$
The total value of the account at retirement will be:
Total value at retirement $=\$ 436,478.52+1,336,476.07$
Total value at retirement $=\$ 1,772,954.59$
This amount is the present value of the annual withdrawals. Now we can use the present value of an annuity equation to find the annuity amount. Doing so, we find the annual withdrawal will be:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
$\$ 1,772,954.59=C\left[\left\{1-[1 /(1+.0675)]^{25}\right\} / .0675\right]$
$C=\$ 148,727.69$

## B-82 SOLUTIONS

45. We need to use the PVA due equation, that is:

PVA $_{\text {due }}=(1+r)$ PVA
Using this equation:
$\mathrm{PVA}_{\text {due }}=\$ 61,000=[1+(.0815 / 12)] \times C\left[\left\{1-1 /[1+(.0815 / 12)]^{60}\right\} /(.0815 / 12)\right.$
$\$ 60,588.50=\$ C\left\{1-\left[1 /(1+.0815 / 12)^{60}\right]\right\} /(.0815 / 12)$
$C=\$ 1,232.87$
Notice, when we find the payment for the PVA due, we simply discount the PV of the annuity due back one period. We then use this value as the PV of an ordinary annuity.
46. a. If the payments are in the form of an ordinary annuity, the present value will be:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
PVA \(=\$ 10,000\left[\left\{1-[1 /(1+.11)]^{5}\right\} / .11\right]\)
PVA \(=\$ 36,958.97\)
```

If the payments are an annuity due, the present value will be:

$$
\begin{aligned}
& \mathrm{PVA}_{\text {due }}=(1+r) \mathrm{PVA} \\
& \mathrm{PVA}_{\text {due }}=(1+.11) \$ 36,958.97 \\
& \mathrm{PVA}_{\text {due }}=\$ 41,024.46
\end{aligned}
$$

b. We can find the future value of the ordinary annuity as:

```
\(\mathrm{FVA}=C\left\{\left[(1+r)^{t}-1\right] / r\right\}\)
\(\mathrm{FVA}=\$ 10,000\left\{\left[(1+.11)^{5}-1\right] / .11\right\}\)
\(\mathrm{FVA}=\$ 62,278.01\)
```

If the payments are an annuity due, the future value will be:
$\mathrm{FVA}_{\text {due }}=(1+r) \mathrm{FVA}$
$\mathrm{FVA}_{\text {due }}=(1+.11) \$ 62,278.01$
$\mathrm{FVA}_{\text {due }}=\$ 69,128.60$
c. Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary annuity will always higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.
47. Here, we need to find the difference between the present value of an annuity and the present value of a perpetuity. The present value of the annuity is:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 8,000\left\{\left[1-(1 / 1.09)^{30}\right] / .09\right\}$
PVA $=\$ 82,189.23$
And the present value of the perpetuity is:
$\mathrm{PVP}=C / r$
$\mathrm{PVP}=\$ 8,000 / .09$
$\mathrm{PVP}=\$ 88,888.89$
So, the difference in the present values is:
Difference $=\$ 88,888.89-82,189.23$
Difference $=\$ 6,699.66$
There is another common way to answer this question. We need to recognize that the difference in the cash flows is a perpetuity of $\$ 8,000$ beginning 31 years from now. We can find the present value of this perpetuity and the solution will be the difference in the cash flows. So, we can find the present value of this perpetuity as:
$\mathrm{PVP}=C / r$
$\mathrm{PVP}=\$ 8,000 / .09$
$\mathrm{PVP}=\$ 88,888.89$
This is the present value 30 years from now, one period before the first cash flows. We can now find the present value of this lump sum as:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$\mathrm{PV}=\$ 88,888.89 /(1+.09)^{30}$
$\mathrm{PV}=\$ 6,699.66$
This is the same answer we calculated before.
48. Here we need to find the present value of an annuity at several different times. The annuity has semiannual payments, so we need the semiannual interest rate. The semiannual interest rate is:

Semiannual rate $=0.11 / 2$
Semiannual rate $=.055$
Now, we can use the present value of an annuity equation. Doing so, we get:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 9,000\left\{\left[1-(1 / 1.055)^{10}\right] / .055\right\}$
$\mathrm{PVA}=\$ 67,838.63$

## B-84 SOLUTIONS

This is the present value one period before the first payment. The first payment occurs nine and onehalf years from now, so this is the value of the annuity nine years from now. Since the interest rate is semiannual, we must also be careful to use the number of semiannual periods. The value of the annuity five years from now is:

```
\(\mathrm{PV}=\mathrm{FV} /(1+r)^{t}\)
\(\mathrm{PV}=\$ 67,838.63 /(1+.055)^{8}\)
\(\mathrm{PV}=\$ 44,203.58\)
```

And the value of the annuity three years from now is:

```
\(\mathrm{PV}=\mathrm{FV} /(1+r)^{t}\)
\(P V=\$ 67,838.63 /(1+.055)^{12}\)
\(\mathrm{PV}=\$ 35,681.87\)
```

And the value of the annuity today is:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$P V=\$ 67,838.63 /(1+.055)^{18}$
$\mathrm{PV}=\$ 25,878.13$
49. Since the first payment is received six years form today and the last payment is received 20 years from now, there are 15 payments. We can use the present value of an annuity formula, which will give us the present value four years from now, one period before the first payment. So, the present value of the annuity in four years is:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
PVA \(=\$ 1,450\left\{\left[1-(1 / 1.09)^{15}\right] / .09\right\}\)
PVA \(=\$ 11,688.00\)
```

And using the present value equation for a lump sum, the present value of the annuity today is:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$P V=\$ 11,688.00 /(1+.09)^{5}$
$P V=\$ 7,596.40$
50. Here, we have an annuity with two different interest rates. To answer this question, we simply need to find the present value in multiple steps. The present value of the last six years payments at an eight percent interest rate is:

```
\(\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
\(\operatorname{PVA}=\$ 2,500\left[\left\{1-1 /[1+(.08 / 12)]^{72}\right\} /(.08 / 12)\right]\)
PVA \(=\$ 142,586.31\)
```

We can now discount this value back to time zero. We must be sure to use the number of months as the periods since interest is compounded monthly. We also need to use the interest rate that applies during the first four years. Doing so, we find:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$P V=\$ 142,586.31 /(1+.11 / 12)^{48}$
$\mathrm{PV}=\$ 92,015.03$
Now we can find the present value of the annuity payments for the first four years. The present value of these payments is:
$\mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
PVA $=\$ 2,500\left[\left\{1-1 /[1+(.11 / 12)]^{48}\right\} /(.11 / 12)\right]$
PVA $=\$ 96,728.55$
So, the total present value of the cash flows is:
$\mathrm{PV}=\$ 92,015.03+96,728.55$
$\mathrm{PV}=\$ 188,743.58$
51. To answer this question we need to find the future value of the annuity, and then find the present value that makes the lump sum investment equivalent. We also need to make sure to use the number of months as the number of periods. So, the future value of the annuity is:

FVA $=C\left\{\left[(1+r)^{t}-1\right] / r\right\}$
FVA $=\$ 1,200\left\{\left[(1+.07 / 12)^{120}-1\right] /(.07 / 12)\right\}$
$\mathrm{FVA}=\$ 207,701.77$
Now we can find the present value that would permit the lump sum investment to be equal to this future value. This investment has annual compounding, so the number of periods is the number of years. So, the present value we would need to deposit is:
$\mathrm{PV}=\mathrm{FV} /(1+r)^{t}$
$\mathrm{PV}=\$ 207,701.77 /(1+.09)^{10}$
$P V=\$ 87,735.47$
52. Here we need to find the present value of a perpetuity at a date before the perpetuity begins. We will begin by find the present value of the perpetuity. Doing so, we find:
$\mathrm{PVP}=C / r$
$\mathrm{PVP}=\$ 2,500 / .0545$
$\mathrm{PVP}=\$ 45,871.56$

## B-86 SOLUTIONS

This is the present value of the perpetuity at year 19, one period before the payments begin. So, using the present value of a lump sum equation to find the value at year 7 , we find:

```
\(\mathrm{PV}=\mathrm{FV} /(1+r)^{t}\)
\(\mathrm{PV}=\$ 45,871.56 /(1+.0545)^{12}\)
\(\mathrm{PV}=\$ 24,265.23\)
```

53. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We need must be careful to use the cash flows of the loan. Using the present value of an annuity equation, we find:

$$
\begin{aligned}
& \mathrm{PVA}=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right) \\
& \$ 20,000=\$ 1,916.67\left[\left\{1-[1 /(1+r)]^{12}\right\} / r\right]
\end{aligned}
$$

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:
$r=.02219$ or $2.219 \%$
This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12 , so the APR is:

```
APR =.02219 × 12
APR = . 2662 or 26.62%
```

And the EAR is:

```
EAR \(=[1+(\text { APR } / m)]^{m}-1\)
EAR \(=[1+.02219]^{12}-1\)
\(\operatorname{EAR}=.3012\) or \(30.12 \%\)
```

54. To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}(1+r)^{t} \\
& \mathrm{FV}=\$ 25,000(1.094)^{3}+\$ 45,000(1.094)^{2}+\$ 65,000 \\
& \mathrm{FV}=\$ 151,591.08
\end{aligned}
$$

Notice, since we are finding the value at Year 5, the cash flow at Year 5 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow. To find the value in Year 10, we simply need to find the future value of this lump sum. Doing so, we find:

```
FV = PV (1 +r)
FV = $151,591.08(1.094) }\mp@subsup{}{}{5
FV = $237,552.86
```

55. The payment for a loan repaid with equal payments is the annuity payment with the loan value as the PV of the annuity. So, the loan payment will be:
```
PVA \(=C\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)\)
\(\$ 75,000=C\left\{\left[1-1 /(1+.09)^{5}\right] / .09\right\}\)
\(C=\$ 29,629.11\)
```

The interest payment is the beginning balance times the interest rate for the period, and the principal payment is the total payment minus the interest payment. The ending balance is the beginning balance minus the principal payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal payment is:
$\left.\begin{array}{crrrrr} & \text { Beginning } & & \text { Total }\end{array} \begin{array}{r}\text { Interest }\end{array} \begin{array}{r}\text { Principal }\end{array} \quad \begin{array}{r}\text { Ending } \\ \text { Year }\end{array} \quad \begin{array}{r}\text { Balance }\end{array} \quad \begin{array}{r}\text { Payment }\end{array} \quad \begin{array}{r}\text { Payment }\end{array}\right)$

In the third year, $\$ 2,446.44$ of interest is paid.
Total interest over life of the loan $=\$ 6,750.00+4,690.88+2,446.44$
Total interest over life of the loan $=\$ 13,887.32$
56. This amortization table calls for equal principal payments of $\$ 15000$ per year. The interest payment is the beginning balance times the interest rate for the period, and the total payment is the principal payment plus the interest payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal principal reduction is:

|  | Beginning |
| :---: | ---: | ---: | ---: | ---: | ---: |$\quad$| Total |
| ---: |$\quad$| Interest |
| ---: | | Principal |
| ---: |$\quad$| Ending |
| ---: |
| Year |$\quad$| Balance |
| ---: |$\quad$| Payment |
| ---: |$\quad$| $\underline{\text { Payment }}$ |
| ---: |

In the third year, $\$ 2,250$ of interest is paid.
Total interest over life of the loan $=\$ 6,750+4,500+2,250$
Total interest over life of the loan $=\$ 13,500$
Notice that the total payments for the equal principal reduction loan are lower. This is because more principal is repaid early in the loan, which reduces the total interest expense over the life of the loan.

## B-88 SOLUTIONS

## Challenge

57. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $\$ 12,000$ you must repay in one year, and the $\$ 10,680$ you borrow today. The interest rate of the loan is:
```
\(\$ 12,000=\$ 10,440(1+r)\)
\(r=(\$ 12,000 / 10,440)-1\)
\(r=.1494\) or \(14.94 \%\)
```

Because of the discount, you only get the use of $\$ 10,440$, and the interest you pay on that amount is $14.94 \%$, not $11 \%$.
58. Here we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:
$\mathrm{APR}=12\left[(1.09)^{1 / 12}-1\right]=8.65 \%$
To find the value today of the back pay from two years ago, we will find the FV of the annuity, and then find the FV of the lump sum. Doing so gives us:

$$
\begin{aligned}
& \mathrm{FVA}=(\$ 44,000 / 12)\left[\left\{[1+(.0865 / 12)]^{12}-1\right\} /(.0865 / 12)\right]=\$ 45,786.76 \\
& \mathrm{FV}=\$ 45,786.76(1.09)=\$ 49,907.57
\end{aligned}
$$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year's back pay:
$\operatorname{FVA}=(\$ 46,000 / 12)\left[\left\{[1+(.0865 / 12)]^{12}-1\right\} /(.0865 / 12)\right]=\$ 47,867.98$
Next, we find the value today of the five year's future salary:
PVA $=(\$ 49,000 / 12)\left\{\left[\left\{1-\left\{1 /[1+(.0865 / 12)]^{12(5)}\right\}\right] /(.0865 / 12)\right\}=\$ 198,332.55\right.$
The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

$$
\text { Award }=\$ 49,907.57+47,867.98+198,332.55+100,000+20,000=\$ 416,108.10
$$

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.
59. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

Loan repayment amount $=\$ 10,000(1.09)=\$ 10,900$
The amount you will receive today is the principal amount of the loan times one minus the points.
Amount received $=\$ 10,000(1-.03)=\$ 9,700$

Now, we simply find the interest rate for this PV and FV.
$\$ 10,900=\$ 9,700(1+r)$
$r=(\$ 10,900 / \$ 9,700)-1=.1237$ or $12.37 \%$
60. We need to find the FV of the premiums to compare with the cash payment promised at age 65 . We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:
$\mathrm{FV}_{1}=\$ 800(1.11)^{5}=\$ 1,384.05$
$\mathrm{FV}_{2}=\$ 800(1.11)^{4}=\$ 1,214.46$
$F V_{3}=\$ 900(1.11)^{3}=\$ 1,230.87$
$\mathrm{FV}_{4}=\$ 900(1.11)^{2}=\$ 1,108.89$
$\mathrm{FV}_{5}=\$ 1,000(1.11)^{1}=\$ 1,110.00$
Value at year six $=\$ 1,384.05+1,214.46+1,230.87+1,108.89+1,110.00+1,000=\$ 7,012.26$
Finding the FV of this lump sum at the child's $65^{\text {th }}$ birthday:
$\mathrm{FV}=\$ 7,012.26(1.07)^{59}=\$ 379,752.76$
The policy is not worth buying; the future value of the deposits is $\$ 379,752.76$, but the policy contract will pay off $\$ 350,000$. The premiums are worth $\$ 29,752.76$ more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:
$\mathrm{PV}=\$ 800 / 1.11+\$ 800 / 1.11^{2}+\$ 900 / 1.11^{3}+\$ 900 / 1.11^{4}+\$ 1,000 / 1.11^{5}+\$ 1,000 / 1.11^{6}$
$\mathrm{PV}=\$ 3,749.04$

And the value today of the $\$ 350,000$ at age 65 is:
$\mathrm{PV}=\$ 350,000 / 1.07^{59}=\$ 6,462.87$
$P V=\$ 6,462.87 / 1.11^{6}=\$ 3,455.31$

The premiums still have the higher cash flow. At time zero, the difference is $\$ 2,148.25$. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $\$ 293.73$, the difference in the cash flows at time zero, for six years at an 11 percent interest rate, and then for 59 years at a seven percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth $\$ 29,752.76$, the difference in the cash flows at time 65!

## Calculator Solutions

1. 

| CFO | $\$ 0$ |
| :--- | :--- |
| $\mathbf{C 0 1}$ | $\$ 800$ |
| F01 | 1 |
| C02 | $\$ 500$ |
| $\mathbf{F 0 2}$ | 1 |
| C03 | $\$ 1,300$ |
| F03 | 1 |
| $\mathbf{C 0 4}$ | $\$ 1,480$ |
| F04 | 1 |
| $\mathrm{I}=10$ |  |
| NPV CPT |  |
| $\$ 3,128.07$ |  |


| CFo | \$0 |
| :---: | :---: |
| C01 | \$800 |
| F01 | 1 |
| C02 | \$500 |
| F02 | 1 |
| C03 | \$1,300 |
| F03 | 1 |
| C04 | \$1,480 |
| F04 | 1 |
| $\mathrm{I}=18$ |  |
| NPV CPT |  |
| \$2,591.65 |  |


| CFO | $\$ 0$ |
| :--- | :--- |
| $\mathbf{C 0 1}$ | $\$ 800$ |
| F01 | 1 |
| C02 | $\$ 500$ |
| $\mathbf{F 0 2}$ | 1 |
| C03 | $\$ 1,300$ |
| F03 | 1 |
| $\mathbf{C 0 4}$ | $\$ 1,480$ |
| $\mathbf{F 0 4}$ | 1 |
| $\mathrm{I}=24$ |  |
| NPV CPT |  |
| $\$ 2,278.18$ |  |

2. 

| Enter | 9 | $6 \%$ |  | $\pm \$ 5,500$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  | $\$ 37,409.31$ |  |  |


| Enter | 5 | $6 \%$ |  | $\pm \$ 8,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} Y$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  | $\$ 33,698.91$ |  |  |


| Enter | 9 | $22 \%$ |  | $\pm \$ 5,500$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  | $\$ 20,824.57$ |  |  |


4.

| Enter | 15 | $9 \%$ |  | $\pm \$ 7,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PV |  |
|  |  | $\$ 56,424.82$ |  | $\mathbf{F V}$ |  |


| Enter | 40 | $9 \%$ |  | $\pm \$ 7,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |


5.

| Enter | 12 | $8.5 \%$ | $\pm \$ 20,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | FV |
| Solve for |  |  |  | $\$ 2,723.06$ |  |

6. 

Enter


| $8.25 \%$ |
| :---: |
| $\mathbf{I} \mathbf{Y}$ |


$\pm \$ 50,000$
PMT
FV
Solve for

B-92 SOLUTIONS
7.

| Enter | 20 | $9.5 \%$ |  | $\pm \$ 3,000$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  |  | $\$ 162,366.70$ |


| Enter | 40 | 9.5\% |  | $\pm$ \$3,000 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  |  | \$1,159,559.98 |
| 8. |  |  |  |  |  |
| Enter | 8 | 5.25\% |  |  | $\pm$ \$30,000 |
|  | N | I/Y | PV | PMT | FV |

9. 

Enter
Solve for

12.

Enter
Solve for
9
NOM
EFF
9.31\%

| Enter | $13 \%$ |  | 12 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for |  | $13.80 \%$ |  |


| Enter | $16 \%$ |  | 365 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | Colve for |
|  |  |  | $17.35 \%$ |
|  |  |  |  |


| Enter | $19 \%$ |  | 2 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for |  | $19.90 \%$ |  |

13. 

Enter
Solve for


10\%
2
EFF
C/Y

| Enter |  | 14\% | 12 |
| :---: | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for | 13.17\% |  |  |
| Enter |  | 9\% | 52 |
|  | NOM | EFF | C/Y |
| Solve for | 8.62\% |  |  |
| Enter |  | 16\% | 365 |
|  | NOM | EFF | C/Y |
| Solve for | 14.85\% |  |  |
| 14. |  |  |  |
| Enter | 12.4\% |  | 12 |
|  | NOM | EFF | C/Y |
| Solve for |  | 13.13\% |  |
| Enter | 12.7\% |  | 2 |
|  | NOM | EFF | C/Y |
| Solve for |  | 13.10\% |  |
| 15. |  |  |  |
| Enter |  | 18\% | 365 |
|  | NOM | EFF | C/Y |
| Solve for | 16.56\% |  |  |

16. 

Enter

Solve for
$13 \times 2=$
$11 \% / 2=$


PMT

17.

Enter

Solve for


PMT


Enter


PMT


Solve for
I/Y
\$7,841.34

| Enter | $20 \times 365=$ | $4.5 \% / 365$ | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\$ 5,000$ <br> $\mathbf{P V}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for |  |  |  | PMT | FV |
| Sol |  |  | $\$ 12,297.33$ |  |  |

18. 

Enter


> PMT
$\pm \$ 75,000$
FV
Solve for
19. $\mathrm{APR}=12(20 \%)=240 \%$

| Enter | $240 \%$ |  | 12 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for |  | $791.61 \%$ |  |

20. 

| Enter | 60 | $8.6 \% / 12=$ | $\pm$ \$69,500 |
| :---: | :---: | :---: | :---: |
|  | N | I/Y | PV |
| Solve for |  |  |  |
| Enter | 8.6\% |  | 12 |
|  | NOM | EFF | C/Y |

21. 

| Enter |  | $1.1 \%$ | $\$ 13,850$ | $\pm \$ 500$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | I/Y | $\mathbf{P V}$ | PMT | FV |
| Solve for | 33.22 |  |  |  |  |

22. 

| Enter | 1 |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PV | PMT |
| Solve for |  | $25.00 \%$ |  |  | FV |

$$
\mathrm{APR}=52(25.00 \%)=1,300.00 \%
$$

| Enter | $1,300 \%$ |  | 52 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | $\mathbf{C / Y}$ |
| Solve for |  | $10,947,544 \%$ |  |

24. 

Enter
Solve for

$\pm \$ 300$
PMT
FV
$\$ 1,048,489.24$
25.


26.

| Enter | $4 \times 4=$ | $.90 \%$ |  | $\pm \$ 2,200$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |

27. 

| CF0 | $\$ 0$ |
| :--- | :--- |
| C01 | $\$ 300$ |
| F01 | 1 |
| C02 | $\$ 900$ |
| F02 | 1 |
| C03 | $\$ 700$ |
| F03 | 1 |
| C04 | $\$ 600$ |
| F04 | 1 |
| I $=9.50$ |  |
| NPV CPT |  |
| $\$ 1,975.08$ |  |

28. 

| CF0 | $\$ 0$ |
| :--- | :--- |
| C01 | $\$ 2,400$ |
| F01 | 1 |
| C02 | $\$ 3,200$ |
| F02 | 1 |
| C03 | $\$ 6,800$ |
| F03 | 1 |
| C04 | $\$ 8,100$ |
| F04 | 1 |
| I $=8.16$ |  |
| NPV CPT |  |
| $\$ 16,247.04$ |  |

B-96 SOLUTIONS
29. First Simple: $\$ 100(.07)=\$ 7 ; 10$ year investment $=\$ 100+10(\$ 7)=\$ 170$

| Enter | 10 |  | $\pm \$ 100$ |  |
| :--- | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ |
|  |  | $5.45 \%$ |  | $\mathbf{F V}$ |

30. $2^{\text {nd }} B G N 2^{\text {nd }}$ SET

| Enter | 60 | $7.45 \% / 12$ | $\pm \$ 48,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} Y$ | $\mathbf{Y V}$ | PMT | FV |
| Solve for |  |  |  | $\$ 954.75$ |  |

31. 

Enter

Solve for
$1.50 \% / 12=$
$\mathbf{I} / \mathbf{Y}$
 PMT


| Enter | 6 | $18 \% / 12=$ | $\pm \$ 10,075.23$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{J / Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  | $\$ 11,016.70$ |  |

$$
\begin{aligned}
& \text { Interest }=\$ 11,016.70-10,000 \\
& \text { Interest }=\$ 1,016.70
\end{aligned}
$$

32. First: $\$ 89,000(.05)=\$ 4,450$ per year
$(\$ 175,000-89,000) / \$ 4,450=19.33$ years
Second:

| Enter |  | $5 \% / 12=$ | $\pm \$ 89,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\$ 175,000$ |
| Solve for | 162.61 |  |  |  |  |

$162.61 / 12=13.55$ years
33.

Enter

Solve for
 PMT

FV
Solve for


PMT
FV
\$1.13

| Enter | 24 |
| :--- | :--- |
| $\mathbf{N}$ |  | \$1.28

34. 

Enter
Solve for

£60 PMT

FV
35.

Enter

Solve for

| 24 |
| :--- |
| $\mathbf{N}$ |

$7 \% / 12=$
$\mathbf{I} / \mathbf{Y}$


FV


$$
\$ 134,010.60+30,000=\$ 164,010.60
$$

36. 

| Enter | 20 | $13 \%$ |  | $\pm \$ 15,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | $\mathbf{F V}$ |

37. 

| Enter | 6 | $\mathbf{I} / \mathbf{Y}$ | $\pm \$ 75,000$ | PMT | \$120,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N |  |  |  | FV |
| Solve for |  | 8.63\% |  |  |  |
| Enter | 13 |  | $\pm \$ 75,000$ |  | \$220,000 |
|  | N | I/Y | PV | PMT | FV |

Solve for
8.15\%
38.

Enter

Solve for


FV
Enter

Solve for

\$77,217.35


FV

Enter

\$50,187.69
Solve for
39.

Enter
$10 \% / 12=$
$\mathbf{I} / \mathbf{Y}$
Solve for

40.

Enter



Solve for .848\%

$$
\mathrm{APR}=.848 \%(12)=10.18 \%
$$

41. 

| CF0 <br> C01 | $\$ 24,000,000$ |
| :--- | :--- |
| F01 | 3 |
| C02 | $\$ 27,000,000$ |
| F02 | 1 |
| C03 | $\$ 25,000,000$ |
| F03 | 1 |
| C04 | $\$ 26,000,000$ |
| F04 | 4 |
| C05 |  |
| F05 |  |
| C06 |  |
| F06 |  |
| C07 |  |
| F07 |  |
| I $=11 \%$ |  |
| NPV CPT |  |
| $\$ 163,141,086.06$ |  |

42. 

| CFo | $\$ 16,000,000$ |
| :--- | :--- |
| C01 | $\$ 18,500,000$ |
| F01 | 1 |
| C02 | $\$ 21,000,000$ |
| F02 | 1 |
| C03 | $\$ 23,500,000$ |
| F03 | 1 |
| C04 | $\$ 23,000,000$ |
| F04 | 1 |
| C05 | $\$ 20,000,000$ |
| F05 | 1 |
| C06 | $\$ 18,000,000$ |
| F06 | 1 |
| C07 | $\$ 20,000,000$ |
| F07 | 1 |
| $\mathrm{I}=11 \%$ |  |
| NPV CPT |  |
| $\$ 113,170,277.46$ |  |

43. 



$$
\mathrm{APR}=0.589 \%(12)=7.07 \%
$$

Enter


Solve for

EFF
7.30\%

12
C/Y
44. Future value of bond account:

| Enter | 10 | $7.5 \%$ | $\pm \$ 150,000$ | $\pm \$ 9,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  |  | $\$ 436,478.52$ |

Future value of stock account:


Annual withdrawal amount:

| Enter | 25 | $6.75 \%$ | $\pm \$ 1,772,954.59$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |

45. $2^{\text {nd }}$ BGN $2^{\text {nd }}$ SET

| Enter | 60 | $8.15 \% / 12$ | $\pm \$ 61,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | $\mathbf{F V}$ |

46. 



$$
2^{\text {nd }} \text { BGN } 2^{\text {nd }} \text { SET }
$$

| Enter | $\frac{5}{\mathbf{N}}$ | $\begin{aligned} & 11 \% \\ & \mathbf{I} / \mathbf{Y} \end{aligned}$ | PV | $\begin{gathered} \pm \$ 10,000 \\ \text { PMT } \end{gathered}$ | FV |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Solve for |  |  | \$41,024.46 |  |  |
| Enter | 5 | 11\% |  | $\pm$ 10,000 |  |
|  | N | I/Y | PV | PMT | FV |

B-100 SOLUTIONS

$$
2^{\text {nd }} B G N 2^{\text {nd }} S E T
$$


47. Present value of annuity:

| Enter | 30 | $9 \%$ |  | $\pm \$ 8,000$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  | $\$ 82,189.23$ |  |  |

And the present value of the perpetuity is:
$\mathrm{PVP}=C / r$
$\mathrm{PVP}=\$ 8,000 / .09$
$\mathrm{PVP}=\$ 88,888.89$

So the difference in the present values is:
Difference $=\$ 88,888.89-82,189.23$
Difference $=\$ 6,699.66$
48. Value at $\mathrm{t}=9$

Enter

| 10 | $11 \% / 2=$ |
| :---: | :---: |
| $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ |

Solve for

Value at $\mathrm{t}=5$

| Enter | $4 \times 2=$ | $11 \% / 2=$ |  |  | $\pm \$ 67,838.63$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | $\begin{gathered} \text { PV } \\ \$ 44,203.58 \end{gathered}$ | PMT | FV |
| Solve for |  |  |  |  |  |
| Value at $\mathrm{t}=3$ |  |  |  |  |  |
| Enter | $6 \times 2=$ | $11 \% / 2=$ |  |  | $\pm$ \$67,838.63 |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$35,681.87 |  |  |

Value today

| Enter | $9 \times 2=$ | $11 \% / 2=$ |  | $\pm \$ 67,838.63$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  | $\$ 25,878.13$ |  |  |

49. Value at $t=5$

| Enter | 15 | $9 \%$ |  | $\pm \$ 1,450$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PV |  |
|  |  | $\$ 11,688.00$ |  | $\mathbf{F V}$ |  |

Value today

| Enter | 5 | 9\% |  |  | $\pm \$ 11,688.00$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$7,596.40 |  |  |

50. Value at $\mathrm{t}=4$

| Enter | $6 \times 12=$ | $8 \% / 12=$ |  | $\pm$ 2,500 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$142,586.31 |  |  |

Value today

| Enter | $4 \times 12=$ | $11 \% / 12=$ |  | $\pm$ 2,500 | $\pm$ 142,586.31 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$188,743.58 |  |  |

51. FV of A


Value to invest in B

53.

Enter
Solve for


FV

$$
\mathrm{APR}=2.219 \%(12)=26.62 \%
$$

| Enter | 26.62\% |  | 12 |
| :---: | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for |  | 30.12\% |  |

54

| CFo <br> C01 | $\$ 0$ |
| :--- | :--- |
| F01 | 1 |
| C02 | $\$ 25,000$ |
| F02 | 1 |
| C03 | $\$ 45,000$ |
| F03 | 1 |
| C04 | $\$ 0$ |
| F04 | 1 |
| C05 | $\$ 65,000$ |
| F05 | 1 |
| $\mathrm{I}=9.4 \%$ |  |
| NPV NFV |  |
| $\$ 151,591.08$ |  |


57.

| Enter | 1 |  | $\pm \$ 10,440$ |  | $\$ 12,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |

58. 

| Enter |  | $9 \%$ | 12 |
| :--- | :---: | :---: | :---: |
|  | NOM | EFF | C/Y |
| Solve for | $8.65 \%$ |  |  |



| Enter | $\mathrm{N}$ | $\frac{9 \%}{\mathbf{I} / \mathbf{Y}}$ | $\frac{ \pm \$ 45,786.76}{\mathbf{P V}}$ | PMT | FV |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Solve for |  |  |  |  | \$49,907.57 |
| Enter | 12 | 8.65\% / 12 = |  | \$46,000 / $12=$ |  |
|  | N | I/Y | PV |  | FV |



$$
\text { Award }=\$ 49,907.57+47,867.98+198,332.55+100,000+20,000=\$ 416,108.10
$$

59. 

| Enter | 1 |  | $\$ 9,700$ |  | $\pm \$ 10,900$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  | $12.37 \%$ |  |  |  |

60. Value at Year 6:

| Enter | 5 | $11 \%$ | $\pm \$ 800$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  |  | $\$ 1,348.05$ |
|  |  |  |  |  |  |
| Enter | 4 | $11 \%$ | $\pm \$ 800$ |  | PV |
| Solve for | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\$ 1,214.46$ |


| Enter | 3 | $11 \%$ | $\pm \$ 900$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} Y$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  |  | $\$ 1,230.87$ |


| Enter | 2 | $11 \%$ | $\pm \$ 900$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  |  | $\$ 1,108.89$ |


| Enter | 1 | $11 \%$ | $\pm \$ 1,000$ |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  |  |  | $\$ 1,110$ |

So, at Year 5, the value is: $\$ 1,348.05+1,214.46+1,230.87+1,108.89+1,100$

$$
+1,000=\$ 7,012.26
$$

## B-104 SOLUTIONS

At Year 65, the value is:


The policy is not worth buying; the future value of the deposits is $\$ 379,752.76$ but the policy contract will pay off $\$ 350,000$.

## CHAPTER 6 INTEREST RATES AND BOND VALUTION

## Answers to Concepts Review and Critical Thinking Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.
2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.
3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?
4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.
5. There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor's standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.
6. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly par.
7. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provide the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.
8. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.
9. Treasury bonds have no credit risk, so a rating is not necessary. Junk bonds often are not rated because there would no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).

## B-106 SOLUTIONS

10. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.
11. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."
12. One measure of liquidity is the bid-ask spread. Liquid instruments have relatively small spreads. Looking at Figure 6.4, the bellwether bond has a spread of one tick; it is one of the most liquid of all investments. Generally, liquidity declines after a bond is issued. Some older bonds, including some of the callable issues, have spreads as wide as six ticks.
13. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.
14. A 100 -year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.
15. $a$. The bond price is the present value when discounting the future cash flows from a bond; YTM is the interest rate used in discounting the future cash flows (coupon payments and principal) back to their present values.
$b$. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount, since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.
c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.
2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise-hence, the price of the bond decreases.

NOTE: Most problems do not explicitly list a par value for bonds. Even though a bond can have any par value, in general, corporate bonds in the United States will have a par value of $\$ 1,000$. We will use this par value in all problems unless a different par value is explicitly stated.
3. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:
$\mathrm{P}=\$ 90\left(\left\{1-[1 /(1+.07)]^{8}\right\} / .07\right)+\$ 1,000\left[1 /(1+.07)^{8}\right]$
$\mathrm{P}=\$ 1,119.43$
We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:
$\mathrm{PVIF}_{R, t}=1 /(1+r)^{t}$
which stands for $\underline{P r e s e n t} \underline{V}$ alue $\underline{I}$ Interest $\underline{\text { Factor }}$
$\operatorname{PVIFA}_{R, t}=\left(\left\{1-[1 /(1+r)]^{t}\right\} / r\right)$
which stands for $\underline{P r e s e n t} \underline{V}$ alue $\underline{I n t e r e s t} \underline{\text { Factor of an } \underline{A} n n u i t y}$
These abbreviations are shorthand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in the remainder of the solutions key. The bond price equation for this problem would be:

$$
\begin{aligned}
& \mathrm{P}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 8}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 8}\right) \\
& \mathrm{P}=\$ 1,119.43
\end{aligned}
$$

## B-108 SOLUTIONS

4. Here, we need to find the YTM of a bond. The equation for the bond price is:
$\mathrm{P}=\$ 1,047.50=\$ 80\left(\mathrm{PVIFA}_{R \%}, 9\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%}, 9\right)$
Notice the equation cannot be solved directly for $R$. Using a spreadsheet, a financial calculator, or trial and error, we find:
$R=\mathrm{YTM}=7.26 \%$
If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be lower than the coupon rate since the bond is a premium bond. That still leaves a lot of interest rates to check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate $\mathrm{YTM}=[$ Annual interest payment $+($ Price difference from par / Years to maturity $)] /$
[(Price + Par value) / 2]
Solving for this problem, we get:
Approximate $\mathrm{YTM}=[\$ 80+(\$ 47.50 / 9)] /[(\$ 1,047.50+1,000) / 2]$
Approximate $\mathrm{YTM}=.0833$ or $8.33 \%$
This is not the exact YTM, but it is close, and it will give you a place to start.
5. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:
$\mathrm{P}=\$ 1,051=C\left(\mathrm{PVIFA}_{6.8 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{6.8 \%, 16}\right)$
Solving for the coupon payment, we get:
$C=\$ 73.33$
The coupon payment is the coupon rate times par value. Using this relationship, we get:
Coupon rate $=\$ 73.33 / \$ 1,000$
Coupon rate $=.0733$ or $7.33 \%$
6. To find the price of this bond, we need to realize that the maturity of the bond is 14 years. The bond was issued one year ago, with 15 years to maturity, so there are 14 years left on the bond. Also, the coupons are semiannual, so we need to use the semiannual interest rate and the number of semiannual periods. The price of the bond is:
$\mathrm{P}=\$ 42.50\left(\mathrm{PVIFA}_{3.95 \%, 28}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.95 \%, 28}\right)$
$\mathrm{P}=\$ 1,050.28$
7. Here, we are finding the YTM of a semiannual coupon bond. The bond price equation is:
$\mathrm{P}=\$ 960=\$ 42\left(\mathrm{PVIFA}_{R \%, 26}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 26}\right)$
Since we cannot solve the equation directly for $R$, using a spreadsheet, a financial calculator, or trial and error, we find:
$R=4.463 \%$
Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:
$\mathrm{YTM}=2 \times 4.463 \%$
$\mathrm{YTM}=8.93 \%$
8. Here, we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:
$\mathrm{P}=\$ 1,070=C\left(\mathrm{PVIFA}_{3.45 \%, 21}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.45 \%, 21}\right)$
Solving for the coupon payment, we get:
$C=\$ 39.24$
Since this is the semiannual payment, the annual coupon payment is:
$2 \times \$ 39.24=\$ 78.48$
And the coupon rate is the coupon rate divided by par value, so:
Coupon rate $=\$ 78.48 / \$ 1,000$
Coupon rate $=.0785$ or $7.85 \%$
9. The approximate relationship between nominal interest rates $(R)$, real interest rates $(r)$, and inflation (h), is:
$R=r+h$
Approximate $r=.08-.047$
Approximate $r=.033$ or $3.30 \%$
The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:
$(1+R)=(1+r)(1+h)$
$(1+.08)=(1+r)(1+.047)$
Exact $r=[(1+.08) /(1+.047)]-1$
Exact $r=.0315$ or $3.15 \%$

## B-110 SOLUTIONS

10. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:
$(1+R)=(1+r)(1+h)$
$R=(1+.032)(1+.043)-1$
$R=.0764$ or $7.64 \%$
11. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:
$(1+R)=(1+r)(1+h)$
$h=[(1+.14) /(1+.09)]-1$
$h=.0459$ or $4.59 \%$
12. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:
$(1+R)=(1+r)(1+h)$
$r=[(1+.13) /(1.041)]-1$
$r=.0855$ or $8.55 \%$
13. This is a note. The lower case " $n$ " beside the maturity denotes it as such. The coupon rate, located in the first column of the quote is $4 \%$. The bid price is:

Bid price $=97: 29=97$ 29/32
Bid price $=97.90625 \% \times \$ 1,000$
Bid price $=\$ 979.0625$
The previous day's ask price is found by:
Previous day's asked price $=$ Today's asked price - Change
Previous day's asked price $=9790 / 32-(-1 / 32)$
Previous day's asked price $=9731 / 32$
The previous day's price in dollars was:
Previous day's dollar price $=97.96875 \% \times \$ 1,000$
Previous day's dollar price $=\$ 979.6875$
14. This is a premium bond because it sells for more than $100 \%$ of face value. The current yield is based on the asked price, so the current yield is:

Current yield $=$ Annual coupon payment $/$ Price
Current yield $=\$ 61.25 / \$ 1,135.625$
Current yield $=.0539$ or $5.39 \%$
The YTM is located under the "ASK YLD" column, so the YTM is 4.78\%.
The bid-ask spread is the difference between the bid price and the ask price, so:
Bid-Ask spread $=113: 18-113: 17$
Bid-Ask spread $=1 / 32$

## Intermediate

15. Here, we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:
$\mathrm{P}=C\left(\mathrm{PVIFA}_{R \%}, t\right)+\$ 1,000\left(\mathrm{PVIF}_{R \sigma_{r}, t}\right)$

$$
\begin{aligned}
& \mathrm{X}: \quad \mathrm{P}_{0}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 13}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 13}\right)=\$ 1,167.15 \\
& \mathrm{P}_{1}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 12}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 12}\right)=\$ 1,158.85 \\
& \mathrm{P}_{3}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 10}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 10}\right)=\$ 1,140.47 \\
& \mathrm{P}_{8}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 5}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 5}\right)=\$ 1,082.00 \\
& \mathrm{P}_{12}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 1}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 1}\right)=\$ 1,018.69 \\
& \mathrm{P}_{13}=\$ 1,000 \\
& \mathrm{Y}: \quad \mathrm{P}_{0}=\$ 70\left(\mathrm{PVIFA}_{9 \%, 13}\right)+\$ 1,000\left(\mathrm{PVIF}_{9 \%, 13}\right)=\$ 850.26 \\
& \mathrm{P}_{1}=\$ 70\left(\mathrm{PVIFA}_{9 \%, 12}\right)+\$ 1,000\left(\mathrm{PVIF}_{9 \%, 12}\right)=\$ 856.79 \\
& \mathrm{P}_{3}=\$ 70\left(\mathrm{PVIFA}_{9 \%, 10}\right)+\$ 1,000\left(\mathrm{PVIF}_{9 \%, 10}\right)=\$ 871.65 \\
& \mathrm{P}_{8}=\$ 70\left(\mathrm{PVIFA}_{9 \%, 5}\right)+\$ 1,000\left(\mathrm{PVIF}_{9 \%, 5}\right)=\$ 922.21 \\
& \mathrm{P}_{12}=\$ 70\left(\mathrm{PVIFA}_{9 \%, 1}\right)+\$ 1,000\left(\mathrm{PVIF}_{9 \%, 1}\right)=\$ 981.65 \\
& \mathrm{P}_{13}=\$ 1,000
\end{aligned}
$$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

## B-112 SOLUTIONS

16. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 7 percent. If the YTM suddenly rises to 9 percent:

$$
\begin{array}{ll}
\mathrm{P}_{\text {Bill }} & =\$ 40\left(\mathrm{PVIFA}_{5 \%, 6}\right)+\$ 1,000\left(\mathrm{PVIF}_{5 \%, 6}\right)
\end{array}=\$ 949.24,
$$

The percentage change in price is calculated as:
Percentage change in price $=($ New price - Original price $) /$ Original price

$$
\begin{aligned}
\Delta \mathrm{P}_{\text {Bill }} \% & =(\$ 949.24-1,000) / \$ 1,000=-0.0508 \text { or }-5.08 \% \\
\Delta \mathrm{P}_{\mathrm{Ted}} \% & =(\$ 828.41-1,000) / \$ 1,000=-0.1716 \text { or }-17.16 \%
\end{aligned}
$$

If the YTM suddenly falls to 5 percent:

$$
\begin{aligned}
\mathrm{P}_{\text {Bill }} & =\$ 40\left(\mathrm{PVIFA}_{3 \%, 6}\right)+\$ 1,000\left(\mathrm{PVIF}_{3 \%, 6}\right) \quad=\$ 1,054.17 \\
\mathrm{P}_{\text {Ted }} & =\$ 40\left(\mathrm{PVIFA}_{3 \%, 40}\right)+\$ 1,000\left(\mathrm{PVIF}_{3 \%, 40}\right)=\$ 1,231.15 \\
\Delta \mathrm{P}_{\text {Bill }} \% & =(\$ 1,054.17-1,000) / \$ 1,000=0.0542 \text { or }+5.42 \% \\
\Delta \mathrm{P}_{\text {Ted }} \% & =(\$ 1,231.15-1,000) / \$ 1,000=0.2311 \text { or }+23.11 \%
\end{aligned}
$$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.
17. Initially, at a YTM of 7 percent, the prices of the two bonds are:

$$
\begin{array}{ll}
\mathrm{P}_{\mathrm{J}} & =\$ 20\left(\mathrm{PVIFA}_{3.5 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.5 \%, 16}\right)=\$ 818.59 \\
\mathrm{P}_{\mathrm{S}} & =\$ 50\left(\mathrm{PVIFA}_{3.5 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.5 \%, 16}\right)=\$ 1,181.41
\end{array}
$$

If the YTM rises from 7 percent to 9 percent:

$$
\begin{array}{ll}
\mathrm{P}_{\mathrm{J}} & =\$ 20\left(\mathrm{PVIFA}_{4.5 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{4.5 \%, 16}\right)=\$ 719.15 \\
\mathrm{P}_{\mathrm{S}} & =\$ 50\left(\mathrm{PVIFA}_{4.5 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{4.5 \%, 16}\right)=\$ 1,056.17
\end{array}
$$

The percentage change in price is calculated as:
Percentage change in price $=($ New price - Original price $) /$ Original price

$$
\begin{array}{lll}
\Delta \mathrm{P}_{\mathrm{J}} \% & =(\$ 719.15-818.59) / \$ 818.59 & =-0.1215 \text { or }-12.15 \% \\
\Delta \mathrm{P}_{\mathrm{s}} \% & =(\$ 1,056.17-1,181.41) / \$ 1,181.41=-0.1060 \text { or }-10.60 \%
\end{array}
$$

If the YTM declines from 7 percent to 5 percent:

$$
\begin{array}{ll}
\mathrm{P}_{\mathrm{J}} & =\$ 20\left(\mathrm{PVIFA}_{2.5 \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{2.5 \%, 16}\right)=\$ 934.72 \\
\mathrm{P}_{\mathrm{S}} & =\$ 50\left(\mathrm{PVIFA}_{2.5 \%}, 16\right)+\$ 1,000\left(\mathrm{PVIF}_{2.5 \%, 16}\right)=\$ 1,326.38 \\
\Delta \mathrm{P}_{\mathrm{J}} \% & =(\$ 934.72-818.59) / \$ 818.59= \\
\Delta \mathrm{P}_{\mathrm{s}} \% & =(\$ 1,326.38-1,181.41) / \$ 1,181.41=0.1419 \text { or }+14.19 \% \\
& =0.1227 \text { or }+12.27 \%
\end{array}
$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.
18. The current yield is:

Current yield $=$ Annual coupon payment $/$ Price
Current yield $=\$ 80 / \$ 930$
Current yield $=0.0860$ or $8.60 \%$
The bond price equation for this bond is:
$\mathrm{P}_{0}=\$ 930=\$ 40\left(\mathrm{PVIFA}_{R \%, 36}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 36}\right)$
Using a spreadsheet, financial calculator, or trial and error we find:
$R=4.39 \%$
This is the semiannual interest rate, so the YTM is:
$\mathrm{YTM}=2 \times 4.39 \%$
$\mathrm{YTM}=8.78 \%$
The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield $=(1+0.0439)^{2}-1$
Effective annual yield $=8.97 \%$
19. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:
$\mathrm{P}=\$ 1,062=\$ 37.50\left(\mathrm{PVIFA}_{R \%, 40}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 40}\right)$
Using a spreadsheet, financial calculator, or trial and error, we find:
$R=3.46 \%$

## B-114 SOLUTIONS

This is the semiannual interest rate, so the YTM is:
$\mathrm{YTM}=2 \times 3.46 \%$
$\mathrm{YTM}=6.92 \%$
20. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so one month has passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest $=\$ 86 / 2 \times 5 / 6$
Accrued interest $=\$ 7.17$
And we calculate the clean price as:
Clean price $=$ Dirty price - Accrued interest
Clean price $=\$ 1,090-7.17$
Clean price $=\$ 1,082.83$
21. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are three months until the next coupon payment, so three months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest $=\$ 75 / 2 \times 3 / 6$
Accrued interest $=\$ 18.75$
And we calculate the dirty price as:
Dirty price $=$ Clean price + Accrued interest
Dirty price $=\$ 865+18.75$
Dirty price $=\$ 883.75$
22. The bond has 10 years to maturity, so the bond price equation is:
$\mathrm{P}=\$ 871.55=\$ 41.25\left(\mathrm{PVIFA}_{R \%, 20}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 20}\right)$
Using a spreadsheet, financial calculator, or trial and error, we find:
$R=5.17 \%$
This is the semiannual interest rate, so the YTM is:
$\mathrm{YTM}=2 \times 5.17 \%$
$\mathrm{YTM}=10.34 \%$

The current yield is the annual coupon payment divided by the bond price, so:

Current yield $=\$ 82.50 / \$ 871.55$
Current yield $=0.0947$ or $9.47 \%$
23. a. The coupon bonds have an $8 \%$ coupon which matches the $8 \%$ required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell $=\$ 30,000,000 / \$ 1,000=30,000$
The number of zero coupon bonds to sell would be:
Price of zero coupon bonds $=\$ 1,000 / 1.08^{20}=\$ 214.55$
Number of zero coupon bonds to sell $=\$ 30,000,000 / \$ 214.55=139,829$
b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment $=30,000(\$ 1,080)=\$ 32,400,000$
The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes: repayment $=139,829(\$ 1,000)=\$ 139,829,714$
c. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: $(30,000)(\$ 80)(1-.35)=\$ 1,560,000$ cash outflow
Note that this is cash outflow since the company is making the interest payment.
For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:
$\mathrm{P}_{1}=\$ 1,000 / 1.08^{19}=\$ 231.71$
The year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part $b$, so:

## B-116 SOLUTIONS

Year 1 interest deduction per bond $=\$ 231.71-214.55=\$ 17.16$
The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in year 1 will be:

Cash flows for zeroes in Year $1=(139,829)(\$ 17.16)(.35)=\$ 840,000$
Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year, even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.
24. The maturity is indeterminate. A bond selling at par can have any length of maturity.
25. The bond asked price is $108: 14$, so the dollar price is:

Percentage price $=10814 / 32=108.4375 \%$
Dollar price $=108.4375 \% \times \$ 1,000$
Dollar price $=\$ 1,084.375$
So the bond price equation is:
$\mathrm{P}=\$ 1,084.375=\$ 32.25\left(\mathrm{PVIFA}_{R \%, 30}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 30}\right)$
Using a spreadsheet, financial calculator, or trial and error, we find:
$R=2.805 \%$
This is the semiannual interest rate, so the YTM is:
$\mathrm{YTM}=2 \times 2.805 \%$
$\mathrm{YTM}=5.61 \%$
26. The coupon rate of the bond is 5.375 percent and the bond matures in 25 years. The bond coupon payments are semiannual, so the asked price is:

$$
\begin{aligned}
& \mathrm{P}=\$ 26.875\left(\mathrm{PVIFA}_{3.62 \%, 50}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.62 \%, 50}\right) \\
& \mathrm{P}=\$ 864.10
\end{aligned}
$$

The bid-ask spread is two ticks. Each tick is $1 / 32$, or .03125 percent of par. We also know the bid price must be less than the asked price, so the bid price is:

Bid price $=\$ 864.10-2(.03125)(10)$
Bid price $=\$ 863.48$
27. Here, we need to find the coupon rate of the bond. The price of the bond is:

Percentage price $=10617 / 32=106.53125 \%$
Dollar price $=106.53125 \% \times \$ 1,000$
Dollar price $=\$ 1,065.3125$
So the bond price equation is:
$\mathrm{P}=\$ 1,065.3125=C\left(\mathrm{PVIFA}_{2.48 \%, 14}\right)+\$ 1,000\left(\mathrm{PVIF}_{2.48 \%, 14}\right)$
Solving for the coupon payment, we get:
$C=\$ 30.38$
Since this is the semiannual payment, the annual coupon payment is:
$2 \times \$ 30.38=\$ 60.76$
And the coupon rate is the coupon rate divided by par value, so:
Coupon rate $=\$ 60.76 / \$ 1,000$
Coupon rate $=.0608$ or $6.08 \%$
28. Here we need to find the yield to maturity. The dollar price of the bond is:

Dollar price $=94.183 \% \times \$ 1,000$
Dollar price $=\$ 941.83$
So, the bond price equation is:
$\mathrm{P}=\$ 941.83=\$ 27\left(\mathrm{PVIFA}_{\mathrm{R} \%, 24}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 24}\right)$
Using a spreadsheet, financial calculator, or trial and error, we find:
$R=3.045 \%$
This is the semiannual interest rate, so the YTM is:
$\mathrm{YTM}=2 \times 3.045 \%$
YTM $=6.09 \%$

## B-118 SOLUTIONS

29. The bond price equation is:
$\mathrm{P}=\$ 35.625\left(\mathrm{PVIFA}_{3.01 \%, 18}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.01 \%, 18}\right)$
$\mathrm{P}=\$ 1,075.92$
The current yield is the annual coupon payment divided by the bond price, so:
Current yield $=\$ 71.25 / \$ 1,075.92$
Current yield $=.0662$ or $6.62 \%$
30. Here, we need to find the coupon rate of the bond. The dollar price of the bond is:

Dollar price $=94.375 \% \times \$ 1,000$
Dollar price $=\$ 943.75$
Now, we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:
$\mathrm{P}=\$ 943.75=C\left(\mathrm{PVIFA}_{3.425 \%, 36}\right)+\$ 1,000\left(\mathrm{PVIF}_{3.425 \%, 36}\right)$
Solving for the coupon payment, we get:
$C=\$ 31.68$
Since this is the semiannual payment, the annual coupon payment is:
$2 \times \$ 31.68=\$ 63.37$
And the coupon rate is the coupon rate divided by par value, so:
Coupon rate $=\$ 63.37 / \$ 1,000$
Coupon rate $=.0634$ or $6.34 \%$

## Challenge

31. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

P: $\quad \mathrm{P}_{0}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 5}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 5}\right)=\$ 1,082.00$
$\mathrm{P}_{1}=\$ 90\left(\mathrm{PVIFA}_{7 \%, 4}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 4}\right)=\$ 1,067.74$
Current yield $=\$ 90 / \$ 1,082.00=.0832$ or $8.32 \%$

The capital gains yield is:
Capital gains yield $=($ New price - Original price $) /$ Original price
Capital gains yield $=(\$ 1,067.74-1,082.00) / \$ 1,082.00=-.0132$ or $-1.32 \%$
The current price of Bond $D$ and the price of Bond $D$ in one year is:
D: $\quad \mathrm{P}_{0}=\$ 50\left(\mathrm{PVIFA}_{7 \%, 5}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 5}\right)=\$ 918.00$
$\mathrm{P}_{1}=\$ 50\left(\mathrm{PVIFA}_{7 \%, 4}\right)+\$ 1,000\left(\mathrm{PVIF}_{7 \%, 4}\right)=\$ 932.26$
Current yield $=\$ 50 / \$ 918.00=.0545$ or $5.45 \%$
Capital gains yield $=(\$ 932.26-918.00) / \$ 918.00=+.0155$ or $+1.55 \%$
All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still $7 \%$, but this return is distributed differently between current income and capital gains.
32. $a$. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:
$\mathrm{P}_{0}=\$ 1,105=\$ 80\left(\mathrm{PVIFA}_{R \%, 10}\right)+\$ 1,000\left(\mathrm{PVIF}_{R \%, 10}\right)$
Using a spreadsheet, financial calculator, or trial and error we find:
$R=\mathrm{YTM}=6.54 \%$
b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:
$\mathrm{P}_{2}=\$ 80\left(\mathrm{PVIFA}_{7.54 \%, 8}\right)+\$ 1,000\left(\mathrm{PVIF}_{7.54 \%, 8}\right)=\$ 1,155.80$
To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were $\$ 80$ each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:
$\mathrm{P}_{0}=\$ 1,105=\$ 80\left(\mathrm{PVIFA}_{R \%, 2}\right)+\$ 1,155.80\left(\mathrm{PVIF}_{R \%, 2}\right)$
Solving for $R$, we get:
$R=\mathrm{HPY}=9.43 \%$
The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

## B-120 SOLUTIONS

## Calculator Solutions

3. 

| Enter | 8 | $7 \%$ |  | $\pm \$ 90$ | $\pm \$ 1,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  |  | $\$ 1,119.43$ |  |  |

4. 

Enter

\$1,000
PMT
FV
Solve for
$7.26 \%$
5.

| Enter | 16 | $6.8 \%$ | $\pm \$ 1,051$ |  | $\$ 1,000$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  |  | $\$ 73.33$ |

$$
\text { Coupon rate }=\$ 73.33 / \$ 1,000
$$

$$
\text { Coupon rate }=.0733 \text { or } 7.33 \%
$$

6. 

Enter

## $7.90 \% / 2=$ <br> I/Y

Solve for

$\pm \$ 85 / 2=$ PMT $\pm \$ 1,000$
7.

Enter


I/Y


PV

\$1,000
FV
Solve for $4.463 \%$
$\mathrm{YTM}=4.463 \% \times 2$
$\mathrm{YTM}=8.93 \%$
8.

| Enter | $10.5 \times 2=$ | $6.9 \% / 2=$ | $\pm \$ 1,070$ |  | $\$ 1,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |
| Solve for |  |  |  | $\$ 39.24$ |  |

$$
\begin{aligned}
& \text { Annual coupon }=\$ 39.24 \times 2 \\
& \text { Annual coupon }=\$ 78.48 \\
& \text { Coupon rate }=\$ 78.48 / \$ 1,000 \\
& \text { Coupon rate }=7.85 \%
\end{aligned}
$$

15. Bond X


| Enter | 12 | 7\% |  | $\pm$ \$90 | $\pm \$ 1,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$1,158.85 |  |  |
| Enter | 10 | 7\% |  | $\pm$ \$90 | $\pm$ 1,000 |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$1,140.47 |  |  |


| Enter | 5 | 7\% |  | $\pm \$ 90$ | $\pm \$ 1,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$1,082.00 |  |  |



| Bond Y |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Enter | 13 | 9\% |  | $\pm$ \$70 | $\pm \$ 1,000$ |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | 850.26 |  |  |


| Enter | 12 | 9\% |  | $\pm$ 70 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$856.79 |  |  |


| Enter | 10 | 9\% |  | $\pm$ \$70 | $\pm \$ 1,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| fe |  |  | \$7 |  |  |


| Enter | 5 | 9\% |  | $\pm$ \$70 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$922. |  |  |


| Enter | 1 | 9\% |  | $\pm$ 70 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | 981. |  |  |

16. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 8 percent. If the YTM suddenly rises to 10 percent:

|  | $\mathrm{P}_{\text {Bill }}$ |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Enter | 6 | $5 \%$ |  | $\pm \$ 40$ | $\pm \$ 1,000$ |
|  |  | $\mathbf{N}$ | $\mathrm{I} / \mathbf{Y}$ | PV | PMT |
| Solve for |  |  |  | $\$ 949.24$ |  |



$$
\begin{aligned}
\Delta \mathrm{P}_{\text {Bill }} \% & =(\$ 949.24-1000) / \$ 1,000=-5.08 \% \\
\Delta \mathrm{P}_{\text {Ted }} \% & =(\$ 828.41-1000) / \$ 1,000=-17.16 \%
\end{aligned}
$$

If the YTM suddenly falls to 6 percent:



$$
\begin{aligned}
& \Delta \mathrm{P}_{\text {Bill }} \%=(\$ 1,054.17-1000) / \$ 1,000=+5.42 \% \\
& \Delta \mathrm{P}_{\text {Ted }} \%=(\$ 1,231.15-1000) / \$ 1,000=+23.11 \%
\end{aligned}
$$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.
17. Initially, at a YTM of 7 percent, the prices of the two bonds are:

| Enter | 16 | 3.5\% |  | $\pm$ 20 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$818.59 |  |  |



If the YTM rises from 7 percent to 9 percent:


$$
\begin{aligned}
& \Delta \mathrm{P}_{\mathrm{J}} \%=(\$ 719.15-818.59) / \$ 818.59=-12.15 \% \\
& \Delta \mathrm{P}_{\mathrm{s}} \%=(\$ 1,056.17-1,181.41) / \$ 1,181.41=-10.60 \%
\end{aligned}
$$

If the YTM declines from 7 percent to 5 percent:


$$
\begin{aligned}
& \Delta \mathrm{P}_{\mathrm{J}} \%=(\$ 934.72-818.59) / \$ 818.59=+14.19 \% \\
& \Delta \mathrm{P}_{\mathrm{S}} \%=(\$ 1,326.38-1,181.41) / \$ 1,181.41=+12.27 \%
\end{aligned}
$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.
18.

| Enter | $18 \times 2=$ | $\pm \$ 930$ | $\$ 80 / 2=$ | $\$ 1,000$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{N} / \mathbf{Y}$ | $\mathbf{P V}$ | PMT | FV |

Solve for
4.39\%

$$
\begin{aligned}
\mathrm{YTM} & =2 \times 4.39 \% \\
\text { YTM } & =8.78 \%
\end{aligned}
$$

## B-124 SOLUTIONS

Effective annual yield:

| Enter | $8.78 \%$ |  | 2 |
| :--- | :---: | :---: | :---: |
| SOM | NOM | EFF | C/Y |
| Solve for |  | $8.97 \%$ |  |

19. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

| Enter | $20 \times 2=$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\pm \$ 1,062$ | $\$ 75 / 2=$ | $\$ 1,000$ |

Solve for 3.46\%

$$
\begin{aligned}
& \text { YTM }=2 \times 3.46 \% \\
& \text { YTM }=6.92 \%
\end{aligned}
$$

22. 

| Enter | $10 \times 2=$ |  | $\pm \$ 871.55$ | $\$ 82.50 / 2$ | $\$ 1,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  | $5.17 \%$ |  |  |  |

$\mathrm{YTM}=2 \times 5.17 \%$
YTM $=10.34 \%$
23. a. The coupon bonds have an $8 \%$ coupon which matches the $8 \%$ required return, so they will sell at par. For the zeroes, the price is:

| Enter | 20 | 8\% |  |  | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
|  |  |  | 4, |  |  |

c. The price of the zeroes in one year will be:

| Enter | 19 | 8\% |  |  | $\pm \$ 1,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$231.71 |  |  |
| 25. |  |  |  |  |  |
| Enter | $15 \times 2=$ |  | $\pm \$ 1084.375$ | \$64.50 / 2 | \$1,000 |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 2.805\% |  |  |  |

$$
\begin{aligned}
\mathrm{YTM} & =2 \times 2.805 \% \\
\mathrm{YTM} & =5.61 \%
\end{aligned}
$$

26. 

| Enter | $25 \times 2=$ | $6.48 \% / 2=$ <br> $\mathbf{N}$ | PV <br>  <br> Solve for |  | $\pm \$ 53.75 / 2$ <br> PMT |
| :--- | :---: | :---: | :---: | :---: | :---: | | $\pm \$ 1,000$ |
| :---: |
| FV |

27. 

| Enter | $7 \times 2=$ | 4.96\% / $2=$ | $\pm \$ 1,065.3125$ |  | \$1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  | \$30.38 |  |

$$
\begin{aligned}
& \text { Annual coupon }=\$ 30.38 \times 2 \\
& \text { Annual coupon }=\$ 60.76 \\
& \text { Coupon rate }=\$ 60.76 / \$ 1,000 \\
& \text { Coupon rate }=.0608 \text { or } 6.08 \%
\end{aligned}
$$

28. 

| Enter | $12 \times 2$ |  | $\pm \$ 941.83$ | $\$ 54 / 2$ | $\$ 1,000$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |
| Solve for |  | $3.045 \%$ |  |  |  |

$$
\begin{aligned}
& \mathrm{YTM}=2 \times 3.045 \% \\
& \mathrm{YTM}=.0609 \text { or } 6.09 \%
\end{aligned}
$$

29. 

| Enter | $9 \times 2$ | 6.02\% / 2 |  | \$71.25 / 2 | \$1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$1,075.92 |  |  |

30. 

| Enter | $18 \times 2$ | 6.85\% / 2 | $\pm$ \$947.35 |  | \$1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  |  | \$31.68 |  |

> Annual coupon $=\$ 31.68 \times 2$
> Annual coupon $=\$ 63.37$
Coupon rate $=\$ 63.37 / \$ 1,000$
Coupon rate $=.0634$ or $6.34 \%$

## B-126 SOLUTIONS

31. 

Bond P
$\mathrm{P}_{0}$

| Enter | 5 | 7\% |  | $\pm$ 90 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | ,082 |  |  |

$\mathrm{P}_{1}$

| Enter | 4 | 7\% |  | $\pm$ \$90 | $\pm \$ 1,000$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |

Current yield $=\$ 90 / \$ 1,082.00=8.32 \%$
Capital gains yield $=(\$ 1,067.64-1,082.00) / \$ 1,082.00=-1.32 \%$
Bond D
$\mathrm{P}_{0}$

| Enter | 5 | 7\% |  | $\pm$ \$50 | $\pm$ 1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  |  | \$918.00 |  |  |
| $\mathrm{P}_{1}$ |  |  |  |  |  |
| Enter | 4 | 7\% |  | $\pm$ \$50 | $\pm$ 1,000 |
|  | N | I/Y | PV | PMT | FV |

Solve for
Current yield $=\$ 50 / \$ 918.00=5.45 \%$
Capital gains yield $=(\$ 932.26-918.00) / \$ 918.00=+1.55 \%$
All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still $8 \%$, but this return is distributed differently between current income and capital gains.
32.
a.

| Enter | 10 |  | $\pm \$ 1,105$ | \$80 | \$1,000 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | I/Y | PV | PMT | FV |
| Solve for |  | 6.54\% |  |  |  |

This is the rate of return you expect to earn on your investment when you purchase the bond.
$b$.
Enter
Solve for
The HPY is:

| Enter | 2 |  | $\pm \$ 1,105$ | $\$ 80$ | $\$ 1,155.80$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Solve for | $\mathbf{N}$ | $\mathbf{I} \mathbf{Y}$ | $\mathbf{P V}$ | $\mathbf{P M T}$ | $\mathbf{F V}$ |

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

## CHAPTER 7 EQUITY MARKETS AND STOCK VALUATION

## Answers to Concepts Review and Critical Thinking Questions

1. The value of any investment depends on its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.
2. Investors believe the company will eventually start paying dividends (or be sold to another company).
3. In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.
4. The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever; that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by the methods of this chapter by applying the general method of valuation. A violation of the second assumption might be a start-up firm that isn't currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method of this chapter.
5. The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.
6. The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.
7. Yes. If the dividend grows at a steady rate, so does the stock price. In other words, the dividend growth rate and the capital gains yield are the same.
8. In a corporate election, you can buy votes (by buying shares), so money can be used to influence or even determine the outcome. Many would argue the same is true in political elections, but, in principle at least, no one has more than one vote.
9. It wouldn't seem to be. Investors who don't like the voting features of a particular class of stock are under no obligation to buy it.
10. Investors buy such stock because they want it, recognizing that the shares have no voting power. Presumably, investors pay a little less for such shares than they would otherwise.
11. Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term and long-term. If this is correct, then the statement is false.
12. A reasonable limit for the growth rate is the growth rate of the economy, which in the U.S. has historically been about 3 to 3.5 percent (after accounting for inflation). As we will see in a later chapter, inflation has historically averaged about 3 percent, so 6 to 6.5 percent (after accounting for inflation) would be a reasonable limit.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The constant dividend growth model is:
$\mathrm{P}_{t}=\mathrm{D}_{t} \times(1+g) /(R-g)$
So, the price of the stock today is:
$\mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)$
$\mathrm{P}_{0}=\$ 2.20(1.04) /(.11-.04)$
$\mathrm{P}_{0}=\$ 32.69$
The dividend at year 4 is the dividend today times the FVIF for the growth rate in dividends and four years, so:
$\mathrm{P}_{3}=\mathrm{D}_{3}(1+g) /(R-g)$
$\mathrm{P}_{3}=\mathrm{D}_{0}(1+\mathrm{g})^{4} /(R-g)$
$\mathrm{P}_{3}=\$ 2.20(1.04)^{4} /(.11-.04)$
$\mathrm{P}_{3}=\$ 36.77$
We can do the same thing to find the dividend in Year 16, which gives us the price in Year 15, so:
$\mathrm{P}_{15}=\mathrm{D}_{15}(1+g) /(R-g)$
$\mathrm{P}_{15}=\mathrm{D}_{0}(1+\mathrm{g})^{16} /(R-g)$
$\mathrm{P}_{15}=\$ 2.20(1.04)^{16} /(.11-.04)$
$\mathrm{P}_{15}=\$ 58.87$

## B-130 SOLUTIONS

There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. So, if we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in three years, and we have already calculated the stock price today. The stock price in three years will be:
$\mathrm{P}_{3}=\mathrm{P}_{0}(1+g)^{3}$
$\mathrm{P}_{3}=\$ 32.69(1+.04)^{3}$
$\mathrm{P}_{3}=\$ 36.77$
And the stock price in 15 years will be:
$\mathrm{P}_{15}=\mathrm{P}_{0}(1+g)^{15}$
$\mathrm{P}_{15}=\$ 32.69(1+.04)^{15}$
$\mathrm{P}_{15}=\$ 58.77$
2. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for $R$. Doing so, we find:
$R=\left(\mathrm{D}_{1} / \mathrm{P}_{0}\right)+g$
$R=(\$ 1.90 / \$ 47.00)+.055$
$R=.0954$ or $9.54 \%$
3. The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

Dividend yield $=D_{1} / \mathrm{P}_{0}$
Dividend yield $=\$ 1.90 / \$ 47.00$
Dividend yield $=.0404$ or $4.04 \%$
The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

Capital gains yield $=5.5 \%$
4. Using the constant growth model, we find the price of the stock today is:
$\mathrm{P}_{0}=\mathrm{D}_{1} /(R-g)$
$\mathrm{P}_{0}=\$ 3.75 /(.12-.055)$
$\mathrm{P}_{0}=\$ 57.69$
5. The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:
$R=$ Dividend yield + Capital gains yield
$R=.034+.063$
$R=.0970$ or $9.70 \%$
6. We know the stock has a required return of 11 percent, and the dividend and capital gains yield are equal, so:

Dividend yield $=1 / 2(.11)$
Dividend yield $=.055=$ Capital gains yield
Now we know both the dividend yield and capital gains yield. The dividend is simply the stock price times the dividend yield, so:
$\mathrm{D}_{1}=.055(\$ 75)$
$\mathrm{D}_{1}=\$ 4.13$
This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:
$\mathrm{D}_{1}=\mathrm{D}_{0}(1+g)$
We can solve for the dividend that was just paid:
$\$ 4.13=\mathrm{D}_{0}(1+.055)$
$\mathrm{D}_{0}=\$ 4.13 / 1.055$
$\mathrm{D}_{0}=\$ 3.91$
7. The price of any financial instrument is the present value of the future cash flows. The future dividends of this stock are an annuity for eight years, so the price of the stock is the present value of an annuity, which will be:
$\mathrm{P}_{0}=\$ 17.00\left(\mathrm{PVIFA}_{11 \%, 8}\right)$
$\mathrm{P}_{0}=\$ 87.48$
8. The price a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember, most preferred stock pays a fixed dividend, so the growth rate is zero. This is a special case of the dividend growth model where the growth rate is zero, or the level perpetuity equation. Using this equation, we find the price per share of the preferred stock is:
$R=\mathrm{D} / \mathrm{P}_{0}$
$R=\$ 5.00 / \$ 84.12$
$R=.0594$ or $5.94 \%$
9. If the company uses straight voting, the board of directors is elected one at a time. You will need to own one-half of the shares, plus one share, in order to guarantee enough votes to win the election. So, the number of shares needed to guarantee election under straight voting will be:

Shares needed $=(300,000$ shares $/ 2)+1$
Shares needed $=150,001$

## B-132 SOLUTIONS

And the total cost to you will be the shares needed times the price per share, or:
Total cost $=150,001 \times \$ 63$
Total cost $=\$ 9,450,063$
If the company uses cumulative voting, the board of directors are all elected at once. You will need $1 /(N+1)$ percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company's stock you need will be:

Percent of stock needed $=1 /(N+1)$
Percent of stock needed $=1 /(4+1)$
Percent of stock needed $=.20$ or $20 \%$
So, the number of shares you need to purchase is:
Number of shares to purchase $=(300,000 \times .20)+1$
Number of shares to purchase $=60,001$
And the total cost to you will be the shares needed times the price per share, or:
Total cost $=60,001 \times \$ 63$
Total cost $=\$ 3,780,063$
10. We need to find the growth rate of dividends. Using the constant growth model, we can solve the equation for $g$. Doing so, we find:
$g=R-\left(\mathrm{D}_{1} / \mathrm{P}_{0}\right)$
$g=.12-(\$ 3.80 / \$ 65)$
$g=.0615$ or $6.15 \%$
11. Here, we have a stock that pays no dividends for 20 years. Once the stock begins paying dividends, it will have the same dividends forever, a preferred stock. We value the stock at that point, using the preferred stock equation. It is important to remember that the price we find will be the price one year before the first dividend, so:
$\mathrm{P}_{19}=\mathrm{D}_{20} / R$
$\mathrm{P}_{19}=\$ 20 / .08$
$\mathrm{P}_{19}=\$ 250.00$
The price of the stock today is simply the present value of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:
$\mathrm{P}_{0}=\$ 250.00 / 1.08^{19}$
$\mathrm{P}_{0}=\$ 57.93$
12. Here, we need to value a stock with two different required returns. Using the constant growth model and a required return of 15 percent, the stock price today is:
$\mathrm{P}_{0}=\mathrm{D}_{1} /(R-g)$
$\mathrm{P}_{0}=\$ 3.05 /(.15-.05)$
$\mathrm{P}_{0}=\$ 30.50$
And the stock price today with a 10 percent return will be:
$\mathrm{P}_{0}=\mathrm{D}_{1} /(R-g)$
$\mathrm{P}_{0}=\$ 3.05 /(.10-.05)$
$\mathrm{P}_{0}=\$ 61.00$
All else held constant, a higher required return means that the stock will sell for a lower price. Also, notice that the stock price is very sensitive to the required return. In this case, the required return fell by $1 / 3$ but the stock price doubled.

## Intermediate

13. Here, we have a stock that pays no dividends for seven years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that general constant dividend growth formula is:
$\mathrm{P}_{t}=\left[\mathrm{D}_{t} \times(1+g)\right] /(R-g)$
This means that since we will use the dividend in Year 7, we will be finding the stock price in Year 6. The dividend growth model is similar to the present value of an annuity and the present value of a perpetuity: The equation gives you the present value one period before the first payment. So, the price of the stock in Year 6 will be:
$\mathrm{P}_{6}=\mathrm{D}_{7} /(R-g)$
$\mathrm{P}_{6}=\$ 9.00 /(.13-.05)$
$\mathrm{P}_{6}=\$ 112.50$
The price of the stock today is simply the PV of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:
$\mathrm{P}_{0}=\$ 112.50 / 1.13^{6}$
$\mathrm{P}_{0}=\$ 54.04$
14. The price of a stock is the PV of the future dividends. This stock is paying four dividends, so the price of the stock is the PV of these dividends discounted at the required return. So, the price of the stock is:
$\mathrm{P}_{0}=\$ 12 / 1.14+\$ 17 / 1.14^{2}+\$ 22 / 1.14^{3}+\$ 27 / 1.14^{4}$
$\mathrm{P}_{0}=\$ 39.87$

## B-134 SOLUTIONS

15. With supernormal dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the supernormal growth period. The stock begins constant growth after the fourth dividend is paid, so we can find the price of the stock at Year 4, when the constant dividend growth begins, as:
$\mathrm{P}_{4}=\mathrm{D}_{4}(1+g) /(R-g)$
$\mathrm{P}_{4}=\$ 2.50(1.05) /(.11-.05)$
$\mathrm{P}_{4}=\$ 43.75$
The price of the stock today is the present value of the first four dividends, plus the present value of the Year 4 stock price. So, the price of the stock today will be:
$\mathrm{P}_{0}=\$ 8.00 / 1.11+\$ 13.00 / 1.11^{2}+\$ 15.00 / 1.11^{3}+\$ 2.50 / 1.11^{4}+\$ 43.75 / 1.11^{4}$
$\mathrm{P}_{0}=\$ 59.19$
16. With supernormal dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the supernormal growth period. The stock begins constant growth after the third dividend is paid, so we can find the price of the stock in Year 3, when the constant dividend growth begins as:
$\mathrm{P}_{3}=\mathrm{D}_{3}(1+g) /(R-g)$
$\mathrm{P}_{3}=\mathrm{D}_{0}\left(1+g_{l}\right)^{3}\left(1+g_{2}\right) /(R-g)$
$\mathrm{P}_{3}=\$ 3.05(1.20)^{3}(1.06) /(.13-.06)$
$\mathrm{P}_{3}=\$ 79.81$
The price of the stock today is the present value of the first three dividends, plus the present value of the Year 3 stock price. The price of the stock today will be:
$\mathrm{P}_{0}=\$ 3.05(1.20) / 1.13+\$ 3.05(1.20)^{2} / 1.13^{2}+\$ 3.05(1.20)^{3} / 1.13^{3}+\$ 79.81 / 1.13^{3}$
$\mathrm{P}_{0}=\$ 65.64$
17. The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:
$\mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)$
$\mathrm{P}_{0}=\$ 7.00(1-.05) /[(.10-(-.05)]$
$\mathrm{P}_{0}=\$ 44.33$
18. We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:
$\mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)$

Solving this equation for the dividend gives us:
$\mathrm{D}_{0}=\mathrm{P}_{0}(R-g) /(1+g)$
$\mathrm{D}_{0}=\$ 72(.11-.065) /(1+.065)$
$\mathrm{D}_{0}=\$ 3.04$
19. The highest dividend yield will occur when the stock price is the lowest. So, using the 52 -week low stock price, the highest dividend yield was:

Dividend yield $=\mathrm{D} / \mathrm{P}_{\text {Low }}$
Dividend yield $=\$ 1.42 / \$ 28.84$
Dividend yield $=.0492$ or $4.92 \%$
The lowest dividend yield occurred when the stock price was the highest, so:
Dividend yield $=\mathrm{D} / \mathrm{P}_{\text {High }}$
Dividend yield $=\$ 1.42 / \$ 37.51$
Dividend yield $=.0379$ or $3.79 \%$
20. With supernormal dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the supernormal growth period. The stock begins constant growth in Year 6, so we can find the price of the stock in Year 5, one year before the constant dividend growth begins as:
$\mathrm{P}_{5}=\mathrm{D}_{6}(1+g) /(R-g)$
$\mathrm{P}_{5}=\mathrm{D}_{0}\left(1+g_{t}\right)^{5}\left(1+g_{2}\right) /(R-g)$
$\mathrm{P}_{5}=\$ 1.20(1.19)^{5}(1.05) /(.11-.05)$
$\mathrm{P}_{5}=\$ 50.11$
The price of the stock today is the present value of the first five dividends, plus the present value of the Year 5 stock price. The price of the stock today will be:
$\mathrm{P}_{0}=\$ 1.20(1.19) / 1.11+\$ 1.20(1.19)^{2} / 1.11^{2}+\$ 1.20(1.19)^{3} / 1.11^{3}+\$ 1.20(1.19)^{4} / 1.11^{4}$

$$
+\$ 1.20(1.19)^{5} / 1.11^{5}+\$ 23.73 / 1.11^{5}
$$

$\mathrm{P}_{0}=\$ 37.17$
According to the constant growth model, the stock seems to be overvalued. In fact, the stock is trading at a price more than twice as large as the price we calculated. The factors that would affect the stock price are both the supernormal growth rate and the long-term growth rate, the length of the supernormal growth, and the required return.
21. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for $R$. Doing so, we find:
$R=\left(\mathrm{D}_{1} / \mathrm{P}_{0}\right)+g$
$R=[\$ 1.28(1+.015) / \$ 31.61]+.015$
$R=0.0561$ or $5.61 \%$

## B-136 SOLUTIONS

The required return depends on the company and the industry. Since Duke Energy is a regulated utility company, there is little room for growth. This is the reason for the relatively high dividend yield. Since the company has little reason to keep retained earnings for new projects, a majority of net income is paid to shareholders in the form of dividends. This may change in the near future with the deregulation of the electricity industry. In fact, the deregulation is probably already affecting the expected growth rate for Duke Energy.
22. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for $R$. Doing so, we find:
$R=\left(\mathrm{D}_{1} / \mathrm{P}_{0}\right)+g$
$R=[\$ 0.72(1-.10) / \$ 79.60]+(-.10)$
$R=-0.0919$ or $-9.19 \%$
Obviously, this number is incorrect. The required return can never be negative. JC Penney investors must believe that the dividend growth rate over the past 10 years is not indicative of future growth in dividends.

For JC Penney, same-store sales had fallen during part of this period, while at the same time industry same store sales had increased. Additionally, JC Penney previously owned its own credit subsidiary that had lost money in recent years. The company also experienced increased competition from Wal-Mart, among others.
23. The annual dividend paid to stockholders is $\$ 0.32$, and the dividend yield is 1 percent. Using the equation for the dividend yield:

Dividend yield = Dividend / Stock price
We can plug the numbers in and solve for the stock price:
$.01=\$ 0.32 / \mathrm{P}_{0}$
$\mathrm{P}_{0}=\$ 0.32 / .01$
$\mathrm{P}_{0}=\$ 32.00$
The dividend yield quoted in the newspaper is rounded. This means the price calculated using the dividend will be slightly different from the actual price. The required return for Tootsie Roll shareholders using the dividend discount model is:
$R=\left(\mathrm{D}_{1} / \mathrm{P}_{0}\right)+g$
$R=[\$ 0.32(1+.07) / \$ 32.43]+.07$
$R=0.0806$ or $8.06 \%$
This number seems low, although we are really not able to determine why as of this point in the book. We will have more to say about this number in a later chapter.
24. We are asked to find the dividend yield and capital gains yield for each of the stocks. All of the stocks have a 16 percent required return, which is the sum of the dividend yield and the capital gains yield. To find the components of the total return, we need to find the stock price for each stock. Using this stock price and the dividend, we can calculate the dividend yield. The capital gains yield for the stock will be the total return (required return) minus the dividend yield.

$$
\begin{array}{ll}
\mathrm{W}: & \mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g) \\
& \mathrm{P}_{0}=\$ 2.80(1.10) /(.16-.10) \\
& \mathrm{P}_{0}=\$ 51.33
\end{array}
$$

Dividend yield $=\mathrm{D}_{1} / \mathrm{P}_{0}$
Dividend yield $=\$ 2.80(1.10) / \$ 51.33$
Dividend yield $=.06$ or $6 \%$
Capital gains yield $=$ Total return - Dividend yield
Capital gains yield $=.16-.06$
Capital gains yield $=.10$ or $10 \%$
X: $\quad \mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)$
$\mathrm{P}_{0}=\$ 2.80 /(.16-.00)$
$\mathrm{P}_{0}=\$ 17.50$
Dividend yield $=\mathrm{D}_{1} / \mathrm{P}_{0}$
Dividend yield $=\$ 2.80 / \$ 17.50$
Dividend yield $=.16$ or $16 \%$
Capital gains yield $=$ Total return - Dividend yield
Capital gains yield $=.16-.16$
Capital gains yield $=.00$ or $0 \%$
$\mathrm{Y}: \quad \mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)$
$\mathrm{P}_{0}=\$ 2.80(1-.05) /[.16-(-.05)]$
$\mathrm{P}_{0}=\$ 12.67$
Dividend yield $=\mathrm{D}_{1} / \mathrm{P}_{0}$
Dividend yield $=\$ 2.80(.95) / \$ 12.67$
Dividend yield $=.21$ or $21 \%$
Capital gains yield $=$ Total return - Dividend yield
Capital gains yield $=.16-.21$
Capital gains yield $=-.05$ or $-5 \%$

## B-138 SOLUTIONS

Z: To find the price of Stock Z, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the supernormal growth period. The stock begins constant growth in Year 3, so we can find the price of the stock in Year 2, one year before the constant dividend growth begins as:
$\mathrm{P}_{2}=\mathrm{D}_{2}(1+g) /(R-g)$
$\mathrm{P}_{2}=\mathrm{D}_{0}\left(1+g_{1}\right)^{2}\left(1+g_{2}\right) /(R-g)$
$\mathrm{P}_{2}=\$ 2.80(1.20)^{2}(1.12) /(.16-.12)$
$\mathrm{P}_{2}=\$ 112.90$
The price of the stock today is the present value of the first three dividends, plus the present value of the Year 3 stock price. The price of the stock today will be:

$$
\begin{aligned}
& \mathrm{P}_{0}=\$ 2.80(1.20) / 1.16+\$ 2.80(1.20)^{2} / 1.16^{2}+\$ 112.90 / 1.16^{2} \\
& \mathrm{P}_{0}=\$ 89.79
\end{aligned}
$$

Dividend yield $=\mathrm{D}_{1} / \mathrm{P}_{0}$
Dividend yield $=\$ 2.80(1.20) / \$ 89.79$
Dividend yield $=.037$ or $3.7 \%$
Capital gains yield $=$ Total return - Dividend yield
Capital gains yield $=.16-.037$
Capital gains yield $=.123$ or $12.3 \%$
In all cases, the required return is $16 \%$, but the return is distributed differently between current income and capital gains. High-growth stocks have an appreciable capital gains component but a relatively small current income yield; conversely, mature, negative-growth stocks provide a high current income but also price depreciation over time.
25. $a$. Using the constant growth model, the price of the stock paying annual dividends will be:
$\mathrm{P}_{0}=\mathrm{D}_{0}(1+g) /(R-g)=\$ 2.40(1.06) /(.12-.06)=\$ 42.40$
b. If the company pays quarterly dividends instead of annual dividends, the quarterly dividend will be one-fourth of annual dividend, or:

Quarterly dividend: \$2.40(1.06)/4 = \$0.636
To find the equivalent annual dividend, we must assume that the quarterly dividends are reinvested at the required return. We can then use this interest rate to find the equivalent annual dividend. In other words, when we receive the quarterly dividend, we reinvest it at the required return on the stock. So, the effective quarterly rate is:

Effective quarterly rate: $1.12^{25}-1=.0287$

The effective annual dividend will be the FVA of the quarterly dividend payments at the effective quarterly required return. In this case, the effective annual dividend will be:

Effective $\mathrm{D}_{1}=\$ 0.636\left(\mathrm{FVIFA}_{2.87 \%, 4}\right)=\$ 2.66$
Now, we can use the constant growth model to find the current stock price as:
$\mathrm{P}_{0}=\$ 2.66 /(.12-.06)=\$ 44.26$
Note that we cannot simply find the quarterly effective required return and growth rate to find the value of the stock. This would assume the dividends increased each quarter, not each year. Assuming you can reinvest the dividends at the required return of the stock, this model would be appropriate.

## CHAPTER 8 <br> NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

## Answers to Concepts Review and Critical Thinking Questions

1. A payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR.
2. If a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus the payback period must be less than the project life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus PI must be greater than 1. If NPV is positive for a certain discount rate R , then it will be zero for some larger discount rate $\mathrm{R}^{*}$; thus the IRR must be greater than the required return.
3. $a$. Payback period is simply the break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that payback before this cutoff, and reject projects that take longer to payback.
b. The worst problem associated with payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.
c. Despite its shortcomings, payback is often used because (1) the analysis is straightforward and simple and (2) accounting numbers and estimates are readily available. Materiality consider-ations often warrant a payback analysis as sufficient; maintenance projects are another example where the detailed analysis of other methods is often not needed. Since payback is biased towards liquidity, it may be a useful and appropriate analysis method for short-term projects where cash management is most important.
4. $a$. The average accounting return is interpreted as an average measure of the accounting performance of a project over time, computed as some average profit measure due to the project divided by some average balance sheet value for the project. This text computes AAR as average net income with respect to average (total) book value. Given some predetermined cutoff for AAR, the decision rule is to accept projects with an AAR in excess of the target measure, and reject all other projects.
b. AAR is not a measure of cash flows and market value, but a measure of financial statement accounts that often bear little semblance to the relevant value of a project. In addition, the selection of a cutoff is arbitrary, and the time value of money is ignored. For a financial manager, both the reliance on accounting numbers rather than relevant market data and the exclusion of time value of money considerations are troubling. Despite these problems, AAR continues to be used in practice because (1) the accounting information is usually available, (2) analysts often use accounting ratios to analyze firm performance, and (3) managerial compensation is often tied to the attainment of certain target accounting ratio goals.
5. a. NPV is simply the sum of the present values of a project's cash flows. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV.
b. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and can differentiate between projects of different scale and time horizon. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and not certain, but this is a problem shared by the other performance criteria as well. A project with NPV $=\$ 2,500$ implies that the total shareholder wealth of the firm will increase by $\$ 2,500$ if the project is accepted.
6. a. The IRR is the discount rate that causes the NPV of a series of cash flows to be equal to zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The IRR decision rule is to accept projects with IRRs greater than the discount rate, and to reject projects with IRRs less than the discount rate.
b. IRR is the interest rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and also ambiguously ranks some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.
c. IRR is frequently used because it is easier for many financial managers and analysts to rate performance in relative terms, such as " $12 \%$ ", than in absolute terms, such as " $\$ 46,000$." IRR may be a preferred method to NPV in situations where an appropriate discount rate is unknown or uncertain; in this situation, IRR would provide more information about the project than would NPV.
7. $a$. The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one.
b. $\mathrm{PI}=(\mathrm{NPV}+$ cost $) /$ cost $=1+(\mathrm{NPV} /$ cost $)$. If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the "bang for the buck" of each particular project.
8. For a project with future cash flows that are an annuity:

Payback $=\mathrm{I} / \mathrm{C}$
And the IRR is:
$0=-\mathrm{I}+\mathrm{C} / \mathrm{IRR}$
Solving the IRR equation for IRR, we get:
$\mathrm{IRR}=\mathrm{C} / \mathrm{I}$
Notice this is just the reciprocal of the payback. So:
IRR $=1 /$ Payback
For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR.
9. There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.
10. The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (NPV, IRR, and profitability index) are really only slightly more difficult in practice.
11. Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the "revenues" from not-for-profit ventures are not tangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!
12. The yield to maturity is the internal rate of return on a bond. The two concepts are identical with the exception that YTM is applied to bonds and IRR is applied to capital budgeting.
13. The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.
14. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is not relevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment "works" only if you use the required return as the reinvestment rate is also irrelevant simply because reinvestment is not relevant in the first place to the value of the project.

One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not "repatriate" the money. Such funds are said to be "blocked" and reinvestment becomes relevant because the cash flows are not truly available.
15. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

|  | $\mathrm{C}_{0}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | IRR |
| :--- | ---: | ---: | ---: | ---: |
| Project A | $-\$ 100$ | $\$ 10$ | $\$ 110$ | $10 \%$ |

Suppose this $\$ 100$ is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

## B-144 SOLUTIONS

## Solutions to Questions and Problems

## Basic

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. To calculate the payback period, we need to find the time that the project has recovered its initial investment. After two years, the project has created:
$\$ 1,200+1,500=\$ 2,700$
in cash flows. The project still needs to create another:
$\$ 3,400-2,700=\$ 700$
in cash flows. During the third year, the cash flows from the project will be $\$ 900$. So, the payback period will be 2 years, plus what we still need to make divided by what we will make during the third year. The payback period is:

Payback $=2+(\$ 700 / \$ 900)$
Payback $=2.78$ years
2. To calculate the payback period, we need to find the time that the project has recovered its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $\$ 3,400$, the payback period is:

Payback $=4+\$ 360 / \$ 760$
Payback $=4.47$ years
There is a shortcut to calculate payback period when the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the $\$ 3,400$ cost, the payback period is:

$$
\begin{aligned}
& \text { Payback }=\$ 3,400 / \$ 760 \\
& \text { Payback }=4.47 \text { years }
\end{aligned}
$$

For an initial cost of $\$ 4,450$, the payback period is:

$$
\text { Payback }=\$ 4,450 / \$ 760
$$

Payback $=5.86$ years
The payback period for an initial cost of $\$ 6,800$ is a little trickier. Notice that the total cash inflows after eight years will be:

Total cash inflows $=8(\$ 760)$
Total cash inflows $=\$ 6,080$

If the initial cost is $\$ 6,800$, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

Payback $=\$ 6,800 / \$ 760$
Payback $=8.95$ years
This answer does not make sense since the cash flows stop after eight years, so again, we must conclude the payback period is never
3. Project A has cash flows of:

Cash flows $=\$ 13,000+19,000$
Cash flows $=\$ 32,000$
during the first two years. The cash flows are still short by $\$ 3,000$ of recapturing the initial investment, so the payback for Project A is:

Payback $=2+(\$ 3,000 / \$ 14,000)$
Payback $=2.21$ years
Project B has cash flows of:
Cash flows $=\$ 18,000+27,000+38,000$
Cash flows $=\$ 83,000$
during the first three years. The cash flows are still short by $\$ 12,000$ of recapturing the initial investment, so the payback for Project B is:

Payback $=3+(\$ 12,000 / \$ 225,000)$
Payback $=3.05$ years
Using the payback criterion and a cutoff of 3 years, accept project $A$ and reject project $B$.
4. Our definition of AAR is the average net income divided by the average book value. The average net income for this project is:

Average net income $=(\$ 1,643,000+1,987,000+1,523,000+1,308,000) / 4$
Average net income $=\$ 1,615,250$
And the average book value is:
Average book value $=(\$ 16,000,000+0) / 2$
Average book value $=\$ 8,000,000$

## B-146 SOLUTIONS

So, the AAR for this project is:
AAR $=$ Average net income / Average book value
AAR $=\$ 1,615,250 / \$ 8,000,000$
$\mathrm{AAR}=.2019$ or $20.19 \%$
5. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:
$0=-\$ 130,000+\$ 68,000 /(1+\mathrm{IRR})+\$ 71,000 /(1+\mathrm{IRR})^{2}+\$ 54,000 /(1+\mathrm{IRR})^{3}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{IRR}=23.65 \%$
Since the cash flows are conventional and the IRR is greater than the required return, we would accept the project.
6. The NPV of a project is the PV of the outflows minus by the PV of the inflows. The equation for the NPV of this project at an 11 percent required return is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 130,000+\$ 68,000 / 1.11+\$ 71,000 / 1.11^{2}+\$ 54,000 / 1.11^{3} \\
& \mathrm{NPV}=\$ 28,730.79
\end{aligned}
$$

At an 11 percent required return, the NPV is positive, so we would accept the project.
The equation for the NPV of the project at a 27 percent required return is:

$$
\begin{aligned}
& \text { NPV }=-\$ 130,000+\$ 68,000 / 1.27+\$ 71,000 / 1.27^{2}+\$ 54,000 / 1.27^{3} \\
& \text { NPV }=-\$ 6,074.35
\end{aligned}
$$

At a 27 percent required return, the NPV is negative, so we would reject the project.
7. The NPV of a project is the PV of the outflows minus by the PV of the inflows. Since the cash inflows are an annuity, the equation for the NPV of this project at an 8 percent required return is:

```
NPV = - $7,200 + $1,700(PVIFA 
NPV = $3,419.71
```

At an 8 percent required return, the NPV is positive, so we would accept the project.
The equation for the NPV of the project at a 24 percent required return is:

```
NPV = - $7,200 + $1,700(PVIFA 24%,9)
NPV = -$1,138.65
```

At a 24 percent required return, the NPV is negative, so we would reject the project.
We would be indifferent to the project if the required return was equal to the IRR of the project, since at that required return the NPV is zero. The IRR of the project is:
$0=-\$ 7,200+\$ 1,700\left(\right.$ PVIFA $\left._{\text {IRR }, 9}\right)$
IRR = . 1848 or $18.48 \%$
8. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:
$0=-\$ 36,000+\$ 14,700 /(1+$ IRR $)+\$ 19,600 /(1+\text { IRR })^{2}+\$ 13,100 /(1+\text { IRR })^{3}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$
\operatorname{IRR}=15.37 \%
$$

9. The NPV of a project is the PV of the outflows minus by the PV of the inflows. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 36,000+14,700+19,600+13,100 \\
& \mathrm{NPV}=\$ 11,400
\end{aligned}
$$

The NPV at a 10 percent required return is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 36,000+\$ 14,700 / 1.10+\$ 19,600 / 1.10^{2}+\$ 13,100 / 1.10^{3} \\
& \mathrm{NPV}=\$ 3,404.21
\end{aligned}
$$

The NPV at a 20 percent required return is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 36,000+\$ 14,700 / 1.20+\$ 19,600 / 1 \cdot 20^{2}+\$ 13,100 / 1.20^{3} \\
& \mathrm{NPV}=-\$ 2,557.87
\end{aligned}
$$

And the NPV at a 30 percent required return is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 36,000+\$ 14,700 / 1.30+\$ 19,600 / 1.30^{2}+\$ 13,100 / 1.30^{3} \\
& \mathrm{NPV}=-\$ 7,132.00
\end{aligned}
$$

Notice that as the required return increases, the NPV of the project decreases. This will always be true for projects with conventional cash flows. Conventional cash flows are negative at the beginning of the project and positive throughout the rest of the project.

## B-148 SOLUTIONS

10. a. The IRR is the interest rate that makes the NPV of the project equal to zero. The equation for the IRR of Project A is:
$0=-\$ 40,000+\$ 24,000 /(1+\mathrm{IRR})+\$ 20,000 /(1+\mathrm{IRR})^{2}+\$ 16,000 /(1+\mathrm{IRR})^{3}+\$ 12,000 /(1+\mathrm{IRR})^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR $=32.98 \%$
The equation for the IRR of Project B is:
$0=-\$ 40,000+\$ 14,000 /(1+$ IRR $)+\$ 18,000 /(1+\text { IRR })^{2}+\$ 22,000 /(1+\mathrm{IRR})^{3}+\$ 26,000 /(1+\mathrm{IRR})^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{IRR}=30.72 \%$
Examining the IRRs of the projects, we see that the $\operatorname{IRR}_{A}$ is greater than the $\operatorname{IRR}_{\mathrm{B}}$, so IRR decision rule implies accepting project A . This may not be a correct decision; however, because the IRR criterion has a ranking problem for mutually exclusive projects. To see if the IRR decision rule is correct or not, we need to evaluate the project NPVs.
b. The NPV of Project A is:
$\mathrm{NPV}_{\mathrm{A}}=-\$ 40,000+\$ 24,000 / 1.11+\$ 20,000 / 1.11^{2}+\$ 16,000 / 1.11^{3}+\$ 12,000 / 1.11^{4}$
$\mathrm{NPV}_{\mathrm{A}}=\$ 17,457.90$
And the NPV of Project B is:
$\mathrm{NPV}_{\mathrm{B}}=-\$ 40,000+\$ 14,000 / 1.11+\$ 18,000 / 1.11^{2}+\$ 22,000 / 1.11^{3}+\$ 26,000 / 1.11^{4}$
$\mathrm{NPV}_{\mathrm{B}}=\$ 20,435.03$
The $\mathrm{NPV}_{\mathrm{B}}$ is greater than the $\mathrm{NPV}_{\mathrm{A}}$, so we should accept Project B.
c. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project. Here, we will subtract the cash flows for Project B from the cash flows of Project A. Once we find these differential cash flows, we find the IRR. The equation for the crossover rate is:

Crossover rate: $0=\$ 10,000 /(1+\mathrm{R})+\$ 2,000 /(1+\mathrm{R})^{2}-\$ 6,000 /(1+\mathrm{R})^{3}-\$ 14,000 /(1+\mathrm{R})^{4}$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\mathrm{R}=22.42 \%$
At discount rates above $22.42 \%$ choose project A; for discount rates below $22.42 \%$ choose project B ; indifferent between A and B at a discount rate of $22.42 \%$.
11. The IRR is the interest rate that makes the NPV of the project equal to zero. The equation to calculate the IRR of Project $X$ is:
$0=-\$ 8,000+\$ 4,300 /(1+$ IRR $)+\$ 2,700 /(1+\text { IRR })^{2}+\$ 3,800 /(1+\text { IRR })^{3}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{IRR}=17.16 \%$
For Project Y, the equation to find the IRR is:
$0=-\$ 8,000+\$ 4,100 /(1+$ IRR $)+\$ 2,775 /(1+\text { IRR })^{2}+\$ 3,950 /(1+\text { IRR })^{3}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{IRR}=16.98 \%$
To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project Y from the cash flows from Project X. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: $0=\$ 200 /(1+\mathrm{R})-\$ 75 /(1+\mathrm{R})^{2}-\$ 150 /(1+\mathrm{R})^{3}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\mathrm{R}=7.36 \%$
The table below shows the NPV of each project for different required returns. Notice that Project Y always has a higher NPV for discount rates below 7.36 percent, and always has a lower NPV for discount rates above 7.36 percent.

| $\underline{\mathrm{R}}$ | $\underline{\$ \mathrm{NPV}_{\underline{\mathrm{X}}}}$ | $\underline{\$ \mathrm{NPV}_{\underline{\mathrm{Y}}}}$ |
| ---: | ---: | ---: |
| $0 \%$ | $2,800.00$ | $2,825.00$ |
| $5 \%$ | $1,826.80$ | $1,833.93$ |
| $10 \%$ | 995.49 | 988.35 |
| $15 \%$ | 279.28 | 260.71 |
| $20 \%$ | -342.59 | -370.37 |
| $25 \%$ | -886.40 | -921.60 |

12. a. The equation for the NPV of the project is:
$\mathrm{NPV}=-\$ 25,000,000+\$ 51,000,000 / 1.10-\$ 9,000,000 / 1.10^{2}=\$ 13,925,619.83$
The NPV is greater than 0 , so we would accept the project.
b. The equation for the IRR of the project is:
$0=-\$ 28 \mathrm{M}+\$ 53 \mathrm{M} /(1+\mathrm{IRR})-\$ 8 \mathrm{M} /(1+\mathrm{IRR})^{2}$
From Descartes' rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:
$\operatorname{IRR}=84.49 \%,-80.49 \%$
When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct, that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.
13. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index at a required return of 10 percent is:

$$
\begin{aligned}
& \mathrm{PI}=\left(\$ 13,000 / 1.10+\$ 8,500 / 1 \cdot 10^{2}+\$ 5,500 / 1.10^{3}\right) / \$ 20,000 \\
& \mathrm{PI}=1.149
\end{aligned}
$$

The equation for the profitability index at a required return of 15 percent is:

$$
\begin{aligned}
& \mathrm{PI}=\left(\$ 13,000 / 1.15+\$ 8,500 / 1.15^{2}+\$ 5,500 / 1.15^{3}\right) / \$ 20,000 \\
& \mathrm{PI}=1.067
\end{aligned}
$$

The equation for the profitability index at a required return of 22 percent is:

$$
\begin{aligned}
& \mathrm{PI}=\left(\$ 13,000 / 1.22+\$ 8,500 / 1.22^{2}+\$ 5,500 / 1.22^{3}\right) / \$ 20,000 \\
& \mathrm{PI}=0.970
\end{aligned}
$$

We would accept the project if the required return were 10 percent or 15 percent since the PI is greater than one. We would reject the project if the required return were 22 percent since the PI is less than one.
14. a. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index for each project is:
$\mathrm{PI}_{\mathrm{I}}=\left(\$ 17,000 / 1.11+\$ 20,000 / 1.11^{2}+\$ 24,000 / 1.11^{3}\right) / \$ 45,000$
$\mathrm{PI}_{\mathrm{I}}=1.091$
$\mathrm{PI}_{\text {II }}=\left(\$ 6,000 / 1.11+\$ 13,000 / 1.11^{2}+\$ 9,000 / 1.11^{3}\right) / \$ 20,000$
$\mathrm{PI}_{\text {II }}=1.127$
The profitability index decision rule implies that we accept project II, since $\mathrm{PI}_{\text {II }}$ is greater than the $\mathrm{PI}_{\mathrm{I}}$.
b. The NPV of each project is:

$$
\begin{aligned}
& \mathrm{NPV}_{\mathrm{I}}=-\$ 45,000+\$ 17,000 / 1.11+\$ 20,000 / 1.11^{2}+\$ 24,000 / 1.11^{3} \\
& \mathrm{NPV}_{\mathrm{I}}=\$ 4,096.36 \\
& \mathrm{NPV}_{\text {II }}=-\$ 20,000+\$ 6,000 / 1.11+\$ 13,000 / 1.11^{2}+\$ 9,000 / 1.11^{3} \\
& \mathrm{NPV}_{\mathrm{II}}=\$ 2,537.22
\end{aligned}
$$

The NPV decision rule implies accepting Project I, since the $\mathrm{NPV}_{\mathrm{I}}$ is greater than the $\mathrm{NPV}_{\mathrm{II}}$.
c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitude of the cash flows for the two projects are of different scale. In this problem, project I is roughly 2 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.
15. $a$. The payback period for each project is:

A: $3+(\$ 162,000 / \$ 480,000)=3.34$ years
B: $\quad 21+(\$ 1,100 / \$ 12,700)=2.09$ years
The payback criterion implies accepting project B , because it pays back sooner than project A .
b. The NPV for each project is:

$$
\begin{array}{ll}
\text { A: } & \quad \mathrm{NPV}=-\$ 325,000+\$ 43,000 / 1.15+\$ 51,000 / 1.15^{2}+\$ 69,000 / 1.15^{3}+\$ 480,000 / 1.15^{4} \\
& \mathrm{NPV}=\$ 70,764.81 \\
\text { B: } & \mathrm{NPV}=-\$ 30,000+\$ 15,800 / 1.15+\$ 13,100 / 1.15^{2}+\$ 12,700 / 1.15^{3}+\$ 9,700 / 1.15^{4} \\
& \mathrm{NPV}=\$ 7,541.08
\end{array}
$$

NPV criterion implies we accept project A because project A has a higher NPV than project B.

## B-152 SOLUTIONS

c. The IRR for each project is:

A: $\quad \$ 325,000=\$ 43,000 /(1+$ IRR $)+\$ 51,000 /(1+\text { IRR })^{2}+\$ 69,000 /(1+\text { IRR })^{3}+\$ 480,000 /(1+\text { IRR })^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$
\begin{aligned}
& \text { IRR }=21.91 \% \\
\text { B: } & \$ 30,000=\$ 15,800 /(1+\mathrm{IRR})+\$ 13,100 /(1+\mathrm{IRR})^{2}+\$ 12,700 /(1+\mathrm{IRR})^{3}+\$ 9,700 /(1+\mathrm{IRR})^{4}
\end{aligned}
$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$
\text { IRR }=28.02 \%
$$

IRR decision rule implies we accept project B because IRR for B is greater than IRR for A.
d. The profitability index for each project is:

A: $\quad \mathrm{PI}=\left(\$ 43,000 / 1.15+\$ 51,000 / 1.15^{2}+\$ 69,000 / 1.15^{3}+\$ 480,000 / 1.15^{4}\right) / \$ 325,000$
$\mathrm{PI}=1.218$
B: $\quad \mathrm{PI}=\left(\$ 15,800 / 1.15+\$ 13,100 / 1.15^{2}+\$ 12,700 / 1.15^{3}+\$ 9,700 / 1.15^{4}\right) / \$ 30,000$
$\mathrm{PI}=1.251$
Profitability index criterion implies accept project B because its PI is greater than project A's.
e. In this instance, the NPV criterion implies that you should accept project A, while payback period, IRR, and the profitability index imply that you should accept project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept project A.
16. a. The IRR for each project is:

M: $\quad \$ 190,000=\$ 75,000 /(1+$ IRR $)+\$ 90,000 /(1+\text { IRR })^{2}+\$ 85,000 /(1+\text { IRR })^{3}+\$ 70,000 /(1+\text { IRR })^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
IRR = 24.94\%
$\mathrm{N}: \quad \$ 300,000=\$ 110,000 /(1+\mathrm{IRR})+\$ 145,000 /(1+\mathrm{IRR})^{2}+\$ 130,000 /(1+\mathrm{IRR})^{3}+95,000 /(1+\mathrm{IRR})^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$
\operatorname{IRR}=22.38 \%
$$

IRR decision rule implies we accept project M because $\operatorname{IRR}$ for M is greater than IRR for N .
b. The NPV for each project is:

$$
\begin{aligned}
\mathrm{M}: & \quad \mathrm{NPV}=-\$ 190,000+\$ 75,000 / 1.15+\$ 90,000 / 1.15^{2}+\$ 85,000 / 1.15^{3}+\$ 70,000 / 1.15^{4} \\
& \mathrm{NPV}=\$ 39,181.93 \\
\mathrm{~N}: & \mathrm{NPV}=-\$ 300,000+\$ 110,000 / 1.15+\$ 145,000 / 1.15^{2}+\$ 130,000 / 1.15^{3}+\$ 95,000 / 1.15^{4} \\
& \mathrm{NPV}=\$ 45,086.67
\end{aligned}
$$

NPV criterion implies we accept project N because project N has a higher NPV than project M .
c. Accept project N since the NPV is higher. IRR cannot be used to rank mutually exclusive projects.
17. a. The profitability index for each project is:

$$
\begin{array}{ll}
\mathrm{Y}: & \mathrm{PI}=\left(\$ 16,000 / 1.12+\$ 13,000 / 1.12^{2}+\$ 15,000 / 1.12^{3}+\$ 11,000 / 1.12^{4}\right) / \$ 35,000 \\
& \mathrm{PI}=1.209 \\
\mathrm{Z}: & \mathrm{PI}=\left(\$ 25,000 / 1.12+\$ 24,000 / 1.12^{2}+\$ 22,000 / 1.12^{3}+\$ 21,000 / 1.12^{4}\right) / \$ 60,000 \\
& \mathrm{PI}=1.174
\end{array}
$$

Profitability index criterion implies accept project Y because its PI is greater than project Z's.
b. The NPV for each project is:

$$
\begin{array}{ll}
\mathrm{Y}: & \mathrm{NPV}=-\$ 35,000+\$ 16,000 / 1 \cdot 12+\$ 13,000 / 1 \cdot 12^{2}+\$ 15,000 / 1 \cdot 12^{3}+\$ 11,000 / 1.12^{4} \\
& \mathrm{NPV}=\$ 7,316.64 \\
\mathrm{Z}: & \mathrm{NPV}=-\$ 60,000+\$ 25,000 / 1 \cdot 12+\$ 24,000 / 1.12^{2}+\$ 22,000 / 1.12^{3}+\$ 21,000 / 1.12^{4} \\
& \mathrm{NPV}=\$ 10,459.13
\end{array}
$$

NPV criterion implies we accept project $Z$ because project $Z$ has a higher NPV than project Y .
c. Accept project Z since the NPV is higher. The profitability index cannot be used to rank mutually exclusive projects.

## B-154 SOLUTIONS

18. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project J from the cash flows from Project I. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: $0=\$ 24,000 /(1+\mathrm{R})+\$ 4,000 /(1+\mathrm{R})^{2}-\$ 15,000 /(1+\mathrm{R})^{3}-\$ 20,000 /(1+\mathrm{R})^{4}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\mathrm{R}=9.65 \%$
At a lower interest rate, project J is more valuable because of the higher total cash flows. At a higher interest rate, project I becomes more valuable since the differential cash flows received in the first two years are larger than the cash flows for project J .
19. If the payback period is exactly equal to the project's life then the IRR must be equal to zero since the project pays back exactly the initial investment. If the project never pays back its initial investment, then the IRR of the project must negative.
20. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

```
NPV = - $487,160 + 170,605 + 189,895 + 150,387 + 135,867
NPV = $159,594
```

If the required return is infinite, future cash flows have no value. Even if the cash flow in one year is $\$ 1$ trillion, at an infinite rate of interest, the value of this cash flow today is zero. So, if the future cash flows have no value today, the NPV of the project is simply the cash flow today. So at an infinite interest rate:

NPV $=-\$ 487,160$
The interest rate that makes the NPV of a project equal to zero is the IRR. The equation for the IRR of this project is:

$$
\begin{aligned}
0=-\$ 487, & 160+170,605 /(1+\mathrm{IRR})+189,895 /(1+\mathrm{IRR})^{2}+150,387 /(1+\mathrm{IRR})^{3} \\
& +135,867 /(1+\mathrm{IRR})^{4}
\end{aligned}
$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{IRR}=13.01 \%$
21. a. The payback period for each project is:

$$
\text { F: } \quad 2+(\$ 25,000 / \$ 80,000)=2.31 \text { years }
$$

G: $3+(\$ 40,000 / \$ 190,000)=3.21$ years
The payback criterion implies accepting project $F$ because it pays back sooner than project $G$. Project G does not meet the minimum payback of three years.
b. The NPV for each project is:

$$
\begin{aligned}
\mathrm{F}: & \\
\mathrm{NPV}= & -\$ 175,000+\$ 85,000 / 1 \cdot 10+\$ 65,000 / 1 \cdot 10^{2}+\$ 80,000 / 1 \cdot 10^{3}+\$ 70,000 / 1.10^{4} \\
& \\
& \mathrm{NPV}=\$ 55,000 / 1.10^{5} \\
\mathrm{G}: & \\
\mathrm{NPV}= & -\$ 275,058.53 \\
& \\
& \\
& \\
\mathrm{NPV}= & \$ 1900,000 / 1.10^{4}+\$ 135,000 / 1.092 .80
\end{aligned}
$$

NPV criterion implies we should accept project G because project G has a higher NPV than project H.
c. Even though project $G$ does not meet the payback period of three years, it does provide the largest increase in shareholder wealth, therefore, choose project G. Payback period should generally be ignored in this situation.
22. The MIRR for the project with all three approaches is:

## Discounting approach:

In the discounting approach, we find the value of all cash outflows to time 0 , while any cash inflows remain at the time at which they occur. So, the discounting the cash outflows to time 0 , we find:

Time 0 cash flow $=-\$ 12,000-\$ 4,300 / 1.10^{5}$
Time 0 cash flow $=-\$ 14,669.96$
So, the MIRR using the discounting approach is:

$$
\begin{aligned}
0= & -\$ 14,669.96+\$ 5,800 /(1+\mathrm{MIRR})+\$ 6,500 /(1+\mathrm{MIRR})^{2}+\$ 6,200 /(1+\mathrm{MIRR})^{3} \\
& +5,100 /(1+\text { MIRR })^{4}
\end{aligned}
$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\operatorname{MIRR}=22.63 \%$

## B-156 SOLUTIONS

## Reinvestment approach:

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project. So, the reinvesting the cash flows to time 5, we find:

Time 5 cash flow $=\$ 5,800\left(1.10^{4}\right)+\$ 6,500\left(1.10^{3}\right)+\$ 6,200\left(1.10^{2}\right)+\$ 5,100(1.10)-\$ 4,300$
Time 5 cash flow $=\$ 25,955.28$
So, the MIRR using the discounting approach is:
$0=-\$ 12,000+\$ 25,955.28 /(1+\text { MIRR })^{5}$
$\$ 25,955.28 / \$ 12,000=(1+\text { MIRR })^{5}$
$\operatorname{MIRR}=(\$ 25,955.28 / \$ 12,000)^{1 / 5}-1$
MIRR $=.1668$ or $16.68 \%$

## Combination approach.

In the combination approach, we find the value of all cash outflows at time 0 , and the value of all cash inflows at the end of the project. So, the value of the cash flows is:

Time 0 cash flow $=-\$ 12,000-\$ 4,300 / 1.10^{5}$
Time 0 cash flow $=-\$ 14,669.96$
Time 5 cash flow $=\$ 5,800\left(1.10^{4}\right)+\$ 6,500\left(1.10^{3}\right)+\$ 6,200\left(1.10^{2}\right)+\$ 5,100(1.10)$
Time 5 cash flow $=\$ 30,255.28$
So, the MIRR using the discounting approach is:

```
\(0=-\$ 14,669.96+\$ 30,255.28 /(1+\text { MIRR })^{5}\)
\(\$ 30,255.28 / \$ 14,669.96=(1+\text { MIRR })^{5}\)
MIRR \(=(\$ 30,255.28 / \$ 14,669.96)^{1 / 5}-1\)
MIRR \(=.1558\) or \(15.58 \%\)
```


## Intermediate

23. With different discounting and reinvestment rates, we need to make sure to use the appropriate interest rate. The MIRR for the project with all three approaches is:

## Discounting approach:

In the discounting approach, we find the value of all cash outflows to time 0 at the discount rate, while any cash inflows remain at the time at which they occur. So, the discounting the cash outflows to time 0 , we find:

Time 0 cash flow $=-\$ 12,000-\$ 4,300 / 1.11^{5}$
Time 0 cash flow $=-\$ 14,551.84$

So, the MIRR using the discounting approach is:

```
\(0=-\$ 14,551.84+\$ 5,800 /(1+\mathrm{MIRR})+\$ 6,500 /(1+\mathrm{MIRR})^{2}+\$ 6,200 /(1+\mathrm{MIRR})^{3}\)
    \(+5,100 /(1+\mathrm{MIRR})^{4}\)
```

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
MIRR $=23.08 \%$

## Reinvestment approach:

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project using the reinvestment rate. So, the reinvesting the cash flows to time 5, we find:

Time 5 cash flow $=\$ 5,800\left(1.08^{4}\right)+\$ 6,500\left(1.08^{3}\right)+\$ 6,200\left(1.08^{2}\right)+\$ 5,100(1.08)-\$ 4,300$
Time 5 cash flow $=\$ 24,518.64$
So, the MIRR using the discounting approach is:

```
\(0=-\$ 12,000+\$ 24,518.64 /(1+\mathrm{MIRR})^{5}\)
\(\$ 24,518.64 / \$ 12,000=(1+\text { MIRR })^{5}\)
\(\operatorname{MIRR}=(\$ 24,518.64 / \$ 12,000)^{1 / 5}-1\)
\(\operatorname{MIRR}=.1536\) or \(15.36 \%\)
```


## Combination approach:

In the combination approach, we find the value of all cash outflows at time 0 using the discount rate, and the value of all cash inflows at the end of the project using the reinvestment rate. So, the value of the cash flows is:

Time 0 cash flow $=-\$ 12,000-\$ 4,300 / 1.11^{5}$
Time 0 cash flow $=-\$ 14,551.84$
Time 5 cash flow $=\$ 5,800\left(1.08^{4}\right)+\$ 6,500\left(1.08^{3}\right)+\$ 6,200\left(1.08^{2}\right)+\$ 5,100(1.08)$
Time 5 cash flow $=\$ 28,818.64$
So, the MIRR using the discounting approach is:

```
\(0=-\$ 14,551.84+\$ 28,818.64 /(1+\text { MIRR })^{5}\)
\(\$ 28,818.64 / \$ 14,551.84=(1+M I R R)^{5}\)
\(\operatorname{MIRR}=(\$ 28,818.64 / \$ 14,551.84)^{1 / 5}-1\)
MIRR \(=.1464\) or \(14.64 \%\)
```


## B-158 SOLUTIONS

24. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project S from the cash flows from Project R. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:
$0=\$ 25,000+\$ 2,000 /(1+\mathrm{R})+\$ 3,000 /(1+\mathrm{R})^{2}-\$ 13,000 /(1+\mathrm{R})^{3}-\$ 26,000 /(1+\mathrm{R})^{4}-\$ 4,000 /(1+\mathrm{R})^{5}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\mathrm{R}=10.75 \%$
The NPV of the projects at the crossover rate must be equal, The NPV of each project at the crossover rate is:

R: $\quad \mathrm{NPV}=-\$ 60,000+\$ 21,000 / 1 \cdot 1075+\$ 24,000 / 1 \cdot 1075^{2}+\$ 17,000 / 1 \cdot 1075^{3}+\$ 11,000 / 1 \cdot 1075^{4}$

$$
+\$ 9,000 / 1.1075^{5}
$$

NPV $=\$ 3,754.83$
S: $\quad \mathrm{NPV}=-\$ 85,000+\$ 19,000 / 1.1075+\$ 21,000 / 1.1075^{2}+\$ 30,000 / 1.1075^{3}+\$ 37,000 / 1.1075^{4}$

$$
+\$ 13,000 / 1.1075^{5}
$$

NPV = \$3,754.83
25. The IRR of the project is:
$\$ 48,000=\$ 26,000 /(1+\mathrm{IRR})+\$ 38,000 /(1+\mathrm{IRR})^{2}$
Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
$\mathrm{R}=20.09 \%$
At an interest rate of 12 percent, the NPV is:

```
NPV = $48,000-$26,000/1.12 2-$38,000/1.12
NPV = -$5,507.65
```

At an interest rate of zero percent, we can add cash flows, so the NPV is:

$$
\begin{aligned}
& \text { NPV }=\$ 48,000-\$ 26,000-\$ 38,000 \\
& \text { NPV }=-\$ 16,000.00
\end{aligned}
$$

And at an interest rate of 24 percent, the NPV is:

$$
\begin{aligned}
& \mathrm{NPV}=\$ 48,000-\$ 26,000 / 1.24^{2}-\$ 38,000 / 1.24^{2} \\
& \mathrm{NPV}=+\$ 2,318.42
\end{aligned}
$$

The cash flows for the project are unconventional. Since the initial cash flow is positive and the remaining cash flows are negative, the decision rule for IRR in invalid in this case. The NPV profile is upward sloping, indicating that the project is more valuable when the interest rate increases.
26. By definition, the profitability index is:

PI = Discounted Value of Future Cash Flows / Initial Cost
But note that the discounted value of future cash flows is just the NPV overstated by the neglected initial costs, so:.

NPV = Discounted Value of Future Cash Flows - Initial Cost
Discounted Value of Future Cash Flows = NPV + Initial Cost
Substituting, we get:
$\mathrm{PI}=(\mathrm{NPV}+$ Initial Cost $) /$ Initial Cost
PI $=$ NPV / Initial Cost +1
PI = NPV Index + 1
NPV Index $=\mathrm{PI}-1$
27. a. To have a payback equal to the project's life, given $C$ is a constant cash flow for N years:
$C=\mathrm{I} / \mathrm{N}$
b. To have a positive NPV, $\mathrm{I}<C\left(\mathrm{PVIFA}_{R \sigma, N}\right)$. Thus, $C>\mathrm{I} /\left(\mathrm{PVIFA}_{R \%, N}\right)$.
c. $\quad$ Benefits $=C\left(\mathrm{PVIFA}_{R \sigma}, N\right)=2 \times \operatorname{costs}=2 \mathrm{I}$
$C=2 \mathrm{I} /\left(\mathrm{PVIFA}_{R \%},{ }_{N}\right)$

## B-160 SOLUTIONS

## Challenge

28. a. Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. If you remember back to the chapter on stock valuation, we presented a formula for valuing a stock with constant growth in dividends. This formula is actually the formula for a growing perpetuity, so we can use it here. The PV of the future cash flows from the project is:

PV of cash inflows $=C_{l} /(R-g)$
PV of cash inflows $=\$ 60,000 /(.13-.06)=\$ 857,142.86$
NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:
NPV of the project $=-\$ 925,000+857,142.86=-\$ 67,857.14$
The NPV is negative, so we would reject the project.
b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:
$0=-\$ 925,000+\$ 60,000 /(.13-g)$
Solving for $g$, we get:
$g=.0651$ or $6.51 \%$
29. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:
$0=\$ 20,000-\$ 26,000 /(1+$ IRR $)+\$ 13,000 /(1+\text { IRR })^{2}$
Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don't remember, the quadratic formula is:
$\mathrm{x}=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
In this case, the equation is:
$x=\frac{-(-26,000) \pm \sqrt{(-26,000)^{2}-4(20,000)(13,000)}}{2(26,000)}$

The square root term works out to be:
$676,000,000-1,040,000,000=-364,000,000$
The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.
30. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

Year 2 cash flow $=\$ 165,000(1.04)=\$ 171,600$
Year 3 cash flow $=\$ 190,000(1.04)=\$ 197,600$
Year 4 cash flow $=\$ 205,000(1.04)=\$ 213,200$
Year 5 cash flow $=\$ 183,000(1.04)=\$ 190,320$
So, the NPV of the project is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 450,000+\$ 171,600 / 1.11^{2}+\$ 197,600 / 1.11^{3}+\$ 213,200 / 1.11^{4}+\$ 190,320 / 1.11^{5} \\
& \mathrm{NPV}=\$ 87,144.93
\end{aligned}
$$

And the IRR of the project is:

$$
\begin{aligned}
0= & -\$ 450,000+\$ 171,600 /(1+\mathrm{IRR})^{2}-\$ 197,600 /(1+\mathrm{IRR})^{3}+\$ 213,200 /(1+\mathrm{IRR})^{4} \\
& +\$ 190,320 /(1+\mathrm{IRR})^{5}
\end{aligned}
$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

$$
\operatorname{IRR}=16.95 \%
$$

While this may look like a MIRR calculation, it is not an MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

## Calculator Solutions

5. 

| CF0 | $-\$ 130,000$ |
| :--- | :--- |
| C01 | $\$ 68,000$ |
| F01 | 1 |
| C02 | $\$ 71,000$ |
| F02 | 1 |
| C03 | $\$ 54,000$ |
| F03 | 1 |

IRR CPT
23.65\%
6.

| CFo | -\$130,000 | CFo | -\$130,000 |
| :---: | :---: | :---: | :---: |
| C01 | \$68,000 | C01 | \$68,000 |
| F01 | 1 | F01 | 1 |
| C02 | \$71,000 | C02 | \$71,000 |
| F02 | 1 | F02 | 1 |
| C03 | \$54,000 | C03 | \$54,000 |
| F03 | 1 | F03 | 1 |
| $\mathrm{I}=11 \%$ |  | $\mathrm{I}=27 \%$ |  |
| NPV CPT |  | NPV CPT |  |
| \$28,370.79 |  | -\$6,074.35 |  |

7. 

| CFo | $-\$ 7,200$ |
| :--- | :--- |
| C01 | $\$ 1,700$ |
| F01 | 9 | | I $=8 \%$ |
| :--- |
| NPV CPT |
| $\$ 3,419.71$ |


| CFo | $-\$ 7,200$ |
| :--- | :--- |
| C01 | $\$ 1,700$ |
| F01 | 9 |

$\mathrm{I}=24 \%$
NPV CPT
$-\$ 1,138.65$

| CFo | $-\$ 7,200$ |
| ---: | :--- |
| C01 | $\$ 1,700$ |
| F01 | 9 |
| IRR CPT |  |
| $18.48 \%$ |  |

8. 

| CFo | $-\$ 36,000$ |
| :--- | :--- |
| C01 | $\$ 14,700$ |
| F01 | 1 |
| C02 | $\$ 19,600$ |
| F02 | 1 |
| C03 | $\$ 13,100$ |
| F03 | 1 |
| IRR CPT |  |
| $15.37 \%$ |  |

9. 

| CFo | $-\$ 36,000$ |
| :--- | :--- |
| C01 | $\$ 14,700$ |
| F01 | 1 |
| C02 | $\$ 19,600$ |
| F02 | 1 |
| C03 | $\$ 13,100$ |
| F03 | 1 |


| CF0 $-\$ 36,000$ <br> C01 $\$ 14,700$ <br> F01 1 <br> C02 $\$ 19,600$ <br> F02 1 <br> C03 $\$ 13,100$ <br> F03 1 <br> $\mathrm{I}=10 \%$  <br> NPV CPT  <br> $\$ 3,404.21$  |
| :--- | :--- |


| CF0 | $-\$ 36,000$ |
| :--- | :--- |
| C01 | $\$ 14,700$ |
| F01 | 1 |
| C02 | $\$ 19,600$ |
| F02 | 1 |
| C03 | $\$ 13,100$ |
| F03 | 1 |
| $\mathrm{I}=20 \%$ |  |
| NPV CPT |  |
| $-\$ 2,557.87$ |  |


| CFo | -\$36,000 |
| :---: | :---: |
| C01 | \$14,700 |
| F01 | 1 |
| C02 | \$19,600 |
| F02 | 1 |
| C03 | \$13,100 |
| F03 | 1 |
| $\mathrm{I}=30 \%$ |  |
| NPV CPT |  |
| -\$7,132.00 |  |

10. 

B-164 SOLUTIONS

11.

| CF (X) |  |  |  |
| :---: | :---: | :---: | :---: |
| CFo | -\$8,000 | CFo | -\$8,000 |
| C01 | \$4,300 | C01 | \$4,300 |
| F01 | 1 | F01 | 1 |
| C02 | \$2,700 | C02 | \$2,700 |
| F02 | 1 | F02 | 1 |
| C03 | \$3,800 | C03 | \$3,800 |
| F03 | 1 | F03 | 1 |
| $\mathrm{I}=0$ |  | $\mathrm{I}=25$ |  |
| NPV CPT |  | NPV CPT |  |
| \$2,800 |  | -\$886.40 |  |
| CF (Y) |  |  |  |
| CFo | -\$8,000 | CFo | -\$8,000 |
| C01 | \$4,100 | C01 | \$4,100 |
| F01 | 1 | F01 | 1 |
| C02 | \$2,775 | C02 | \$2,775 |
| F02 | 1 | F02 | 1 |
| C03 | \$3,950 | C03 | \$3,950 |
| F03 | 1 | F03 | 1 |
| $\mathrm{I}=0$ |  | $\mathrm{I}=25$ |  |
| NPV CPT |  | NPV CPT |  |
| \$2,825 |  | -\$921.60 |  |

Crossover rate:

| CFo | $\$ 0$ |
| :--- | :--- |
| C01 | $\$ 200$ |
| - F01 | 1 |
| C02 | $-\$ 75$ |
| F02 | 1 |
| C03 | $-\$ 150$ |
| F03 | 1 |
| CPT IRR |  |
| $7.36 \%$ |  |

12. 



NOTE: This is the only IRR the BA II Plus will calculate. The second IRR of $-80.49 \%$ must be calculated using another program, by hand, or trial and error.
13.

| CFo | \$0 | CFo | \$0 | CFo | \$0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C01 | \$13,000 | C01 | \$13,000 | C01 | \$13,000 |
| F01 | 1 | F01 | 1 | F01 |  |
| C02 | \$8,500 | C02 | \$8,500 | C02 | \$8,500 |
| F02 | 1 | F02 | 1 | F02 | 1 |
| C03 | \$5,500 | C03 | \$5,500 | C03 | \$5,500 |
| F03 | 1 | F03 | 1 | F03 | 1 |
| $\mathrm{I}=10$ |  | $\mathrm{I}=15$ |  | $\mathrm{I}=22$ |  |
| NPV CPT |  | NPV CPT |  | NPV CPT |  |
| \$22,975.21 |  | \$21,347.91 |  | \$19,395.46 |  |

@ $10 \%$ : PI $=\$ 22,975.21 / \$ 20,000=1.149$
@ 15\%: PI $=\$ 21,347.91 / \$ 20,000=1.067$
@ $22 \%$ : PI $=\$ 19,395.46 / \$ 20,000=0.970$
14. $a$. The profitability indexes are:

|  | CF (I) |  | CF (II) |
| :---: | :---: | :---: | :---: |
| CFo | \$0 | CFo | \$0 |
| C01 | \$17,000 | C01 | \$6,000 |
| F01 | 1 | F01 | 1 |
| C02 | \$20,000 | C02 | \$13,000 |
| F02 | 1 | F02 | 1 |
| C03 | \$24,000 | C03 | \$9,000 |
| F03 | 1 | F03 | 1 |
| $\mathrm{I}=11$ |  | $\mathrm{I}=11$ |  |
| NPV CPT |  | NPV CPT |  |
| \$49,096.36 |  | \$22,537.22 |  |

$\mathrm{PI}_{\mathrm{I}}=\$ 49,096.36 / \$ 45,000=1.091$
$\mathrm{PI}_{\mathrm{II}}=\$ 22,537.22 / \$ 20,000=1.127$
b. The NPV of each project is:

CF (I)

| CFo | $-\$ 45,000$ |  | CFo |
| :--- | :--- | :--- | :--- |
| C01 | $\$ 17,000$ |  | $-\$ 20,000$ |
| C01 | C01 | $\$ 6,000$ |  |
| F01 | 1 | F01 | 1 |
| C02 | $\$ 20,000$ |  | C02 |
| F02 | 1 | $\$ 13,000$ |  |
| C03 | $\$ 24,000$ | F02 | 1 |
| C03 | C03 | $\$ 9,000$ |  |
| F | 1 | F03 | 1 |
| I $=11$ |  | I $=11$ |  |
| NPV CPT |  | NPV CPT |  |
| $\$ 4,096.36$ | $\$ 2,537.22$ |  |  |

15. 

| CF (A) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| CFo | -\$325,000 | CFo | -\$325,000 | CFo | \$0 |
| C01 | \$43,000 | C01 | \$43,000 | C01 | \$43,000 |
| F01 | 1 | F01 | 1 | F01 | 1 |
| C02 | \$51,000 | C02 | \$51,000 | C02 | \$51,000 |
| F02 | 1 | F02 | 1 | F02 | 1 |
| C03 | \$69,000 | C03 | \$69,000 | C03 | \$69,000 |
| F03 |  | F03 | 1 | F03 | 1 |
| C04 | \$480,000 | C04 | \$480,000 | C04 | \$480,000 |
| F04 | 1 | F04 | 1 | F04 | 1 |
| $\mathrm{I}=15$ |  | IRR CPT |  | $\mathrm{I}=15$ |  |
| NPV CPT |  | 21.91\% |  | NPV CPT |  |
| \$70,764.81 |  |  |  | \$395,764.81 |  |

$\mathrm{PI}=\$ 395,764.81 / \$ 325,000=1.218$

| $C F(B)$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| CF0 | $-\$ 30,000$ | CF0 | $-\$ 30,000$ | CFo | $\$ 0$ |  |
| C01 | $\$ 15,800$ | C01 | $\$ 15,800$ | C01 | $\$ 15,800$ |  |
| F01 | 1 | F01 | 1 | F01 | 1 |  |
| C02 | $\$ 13,100$ |  | C02 | $\$ 13,100$ |  | C02 |$\$ 13,100$

$\mathrm{PI}=\$ 37,541.08 / \$ 30,000=1.251$
16.

17.



$$
\mathrm{PI}=\$ 70,459.13 / \$ 60,000=1.174
$$

18. 

| CFo | $\$ 0$ |
| :--- | :--- |
| C01 | $\$ 24,000$ |
| F01 | 1 |
| C02 | $\$ 4,000$ |
| F02 | 1 |
| C03 | $-\$ 15,000$ |
| F03 | 1 |
| C04 | $-\$ 20,000$ |
| F04 | 1 |
| CPT IRR |  |
| $9.6478 \%$ |  |

20. 

| CFo | -\$487,160 | CFo | -\$487,160 |
| :---: | :---: | :---: | :---: |
| C01 | \$170,605 | C01 | \$170,605 |
| F01 | 1 | F01 | 1 |
| C02 | \$189,895 | C02 | \$189,895 |
| F02 | 1 | F02 | 1 |
| C03 | \$150,387 | C03 | \$150,387 |
| F03 | 1 | F03 | 1 |
| C04 | \$135,867 | C04 | \$135,867 |
| F04 | 1 | F04 | 1 |
| $\mathrm{I}=0$ |  | IRR CPT |  |
| NPV CPT |  | 13.01\% |  |
| \$139,594 |  |  |  |

21. $b$.

| Project $F$ |  | Project $G$ |  |  |
| :--- | :--- | :--- | :--- | :---: |
| CFo | $-\$ 175,000$ | CFo | $-\$ 275,000$ |  |
| C01 | $\$ 85,000$ | C01 | $\$ 55,000$ |  |
| F01 | 1 | F01 | 1 |  |
| C02 | $\$ 65,000$ | C02 | $\$ 70,000$ |  |
| F02 | 1 | F02 | 1 |  |
| C03 | $\$ 80,000$ | C03 | $\$ 110,000$ |  |
| F03 | 1 | F03 | 1 |  |
| C04 | $\$ 70,000$ | C04 | $\$ 190,000$ |  |
| F04 | 1 | F04 | 1 |  |
| C05 | $\$ 55,000$ | C05 | $\$ 135,000$ |  |
| F05 |  | F05 |  |  |
| I $=10$ |  | I $=10$ |  |  |
| NPV CPT |  | NPV CPT |  |  |
| $\$ 98,058.53$ |  | $\$ 129,092.80$ |  |  |

24. Crossover rate:

| CFo | $\$ 25,000$ |
| :--- | :--- |
| $\mathbf{C 0 1}$ | $\$ 2,000$ |
| F01 | 1 |
| C02 | $\$ 3,000$ |
| F02 | 1 |
| C03 | $-\$ 13,000$ |
| F03 | 1 |
| C04 | $-\$ 26,000$ |
| F04 | 1 |
| C05 | $-\$ 4,000$ |
| F05 | 1 |
| IRR CPT |  |
| $10.75 \%$ |  |


| Project $R$ |  |
| :--- | :--- |
| CFo | $-\$ 60,000$ |
| C01 | $\$ 21,000$ |
| F01 | 1 |
| C02 | $\$ 24,000$ |
| F02 | 1 |
| C03 | $\$ 17,000$ |
| F03 | 1 |
| C04 | $\$ 11,000$ |
| F04 | 1 |
| C05 | $\$ 9,000$ |
| F05 | 1 |
| I $=10.75 \%$ |  |
| NPV CPT |  |
| $\$ 3,754.83$ |  |

Project $S$
CFo
C01
F01
C02
F02
C03
F03
C04
F04
C05
F05
$\mathrm{I}=10.75 \%$
NPV CPT
\$3,754.83
-\$85,000
\$19,000
1
\$21,000
1
$\$ 30,000$
1
\$37,000
1
\$13,000

## B-170 SOLUTIONS

25. 

| CF0 | $\$ 48,000$ |
| :--- | :--- |
| C01 | $-\$ 26,000$ |
| F01 | 1 |
| C02 | $-\$ 38,000$ |
| F02 | 1 |
| IRR CPT |  |
| 20.09\% |  |


| CF0 | $\$ 48,000$ | CF0 | $\$ 48,000$ | CF0 | $\$ 48,000$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| C01 | $-\$ 26,000$ | C01 | $-\$ 26,000$ | C01 | $-\$ 26,000$ |
| F01 | 1 | F01 | 1 | F01 | 1 |
| C02 | $-\$ 38,000$ | C02 | $-\$ 38,000$ | C02 | $-\$ 38,000$ |
| F02 | 1 | F02 | 1 | F02 | 1 |
| I $=0$ |  | I $=12$ |  | I $=24$ |  |
| NPV CPT | NPV CPT | NPV CPT |  |  |  |
| $-\$ 16,000$ | $-\$ 5,507.65$ | $\$ 2,318.42$ |  |  |  |

## CHAPTER 9 <br> MAKING CAPITAL INVESTMENT DECISIONS

## Answers to Concepts Review and Critical Thinking Questions

1. In this context, an opportunity cost refers to the value of an asset or other input that will be used in a project. The relevant cost is what the asset or input is actually worth today, not, for example, what it cost to acquire.
2. For tax purposes, a firm would choose MACRS because it provides for larger depreciation deductions earlier. These larger deductions reduce taxes, but have no other cash consequences. Notice that the choice between MACRS and straight-line is purely a time value issue; the total depreciation is the same, only the timing differs.
3. It's probably only a mild over-simplification. Current liabilities will all be paid presumably. The cash portion of current assets will be retrieved. Some receivables won't be collected, and some inventory will not be sold, of course. Counterbalancing these losses is the fact that inventory sold above cost (and not replaced at the end of the project's life) acts to increase working capital. These effects tend to offset.
4. Management's discretion to set the firm's capital structure is applicable at the firm level. Since any one particular project could be financed entirely with equity, another project could be financed with debt, and the firm's overall capital structure remain unchanged, financing costs are not relevant in the analysis of a project's incremental cash flows according to the stand-alone principle.
5. Depreciation is a non-cash expense, but it is tax-deductible on the income statement. Thus depreciation causes taxes paid, an actual cash outflow, to be reduced by an amount equal to the depreciation tax shield $\mathrm{T}_{\mathrm{C}} \mathrm{D}$. A reduction in taxes that would otherwise be paid is the same thing as a cash inflow, so the effects of the depreciation tax shield must be added in to get the total incremental aftertax cash flows.
6. There are two particularly important considerations. The first is erosion. Will the essentialized book simply displace copies of the existing book that would have otherwise been sold? This is of special concern given the lower price. The second consideration is competition. Will other publishers step in and produce such a product? If so, then any erosion is much less relevant. A particular concern to book publishers (and producers of a variety of other product types) is that the publisher only makes money from the sale of new books. Thus, it is important to examine whether the new book would displace sales of used books (good from the publisher's perspective) or new books (not good). The concern arises any time that there is an active market for used product.
7. Definitely. The damage to Porsche's reputation is definitely a factor the company needed to consider. If the reputation was damaged, the company would have lost sales of its existing car lines.

## B-172 SOLUTIONS

8. One company may be able to produce at lower incremental cost or market better. Also, of course, one of the two may have made a mistake!
9. Porsche would recognize that the outsized profits would dwindle as more products come to market and competition becomes more intense.
10. With a sensitivity analysis, one variable is examined over a broad range of values. With a scenario analysis, all variables are examined for a limited range of values.
11. It is true that if average revenue is less than average cost, the firm is losing money. This much of the statement is therefore correct. At the margin, however, accepting a project with a marginal revenue in excess of its marginal cost clearly acts to increase operating cash flow. At the margin, even if a firm is losing money, as long as marginal revenue exceeds marginal cost, the firm will lose less than it would without the new project.
12. The implication is that they will face hard capital rationing.
13. Forecasting risk is the risk that a poor decision is made because of errors in projected cash flows. The danger is greatest with a new project because the cash flows are usually harder to predict.
14. The option to abandon reflects our ability to reallocate assets if we find our initial estimates were too optimistic. The option to expand reflects our ability to increase cash flows from a project if we find our initial estimates were too pessimistic. Since the option to expand can increase cash flows and the option to abandon reduces losses, failing to consider these two options will generally lead us to underestimate a project's NPV.

## Solutions to Questions and Problems

## Basic

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. The $\$ 8$ million acquisition cost of the land six years ago is a sunk cost. The $\$ 10.2$ million current aftertax value of the land is an opportunity cost if the land is used rather than sold off. The $\$ 24$ million cash outlay and $\$ 900,000$ grading expenses are the initial fixed asset investments needed to get the project going. Therefore, the proper year zero cash flow to use in evaluating this project is

Cash flow $=\$ 10,200,000+24,000,000+900,000$
Cash flow $=\$ 35,100,000$
2. Sales due solely to the new product line are:
$19,000(\$ 21,000)=\$ 399,000,000$

Increased sales of the motor home line occur because of the new product line introduction; thus:
$2,500(\$ 65,000)=\$ 162,500,000$
in new sales is relevant. Erosion of luxury motor coach sales is also due to the new mid-size campers; thus:
$900(\$ 105,000)=\$ 95,500,000$ loss in sales
is relevant. The net sales figure to use in evaluating the new line is thus:
Net sales $=\$ 399,000,000+162,500,000-94,500,000$
Net sales $=\$ 467,000,000$
3. We need to construct a basic income statement. The income statement is:

| Sales | $\$ 860,000$ |  |
| :--- | ---: | ---: |
| Variable costs | 516,000 |  |
| Fixed costs | 195,000 |  |
| Depreciation | 86,000 |  |
| EBIT | $\$ 3,000$ |  |
| Taxes@35\% | 22,050 |  |
| Net income | $\$ 40,950$ |  |
|  |  |  |

4. To find the OCF, we need to complete the income statement as follows:

| Sales | $\$ 734,800$ |
| :--- | ---: | ---: |
| Variable costs | 327,600 |
| Depreciation | 102,000 |
| EBIT | $\$ 305,200$ |
| Taxes@35\% | 106,820 |
| Net income | $\underline{\$ 198,380}$ |

The OCF for the company is:
OCF $=$ EBIT + Depreciation - Taxes
OCF $=\$ 305,200+102,000-106,820$
$\mathrm{OCF}=\$ 300,380$
The depreciation tax shield is the depreciation times the tax rate, so:
Depreciation tax shield $=\mathrm{T}_{\mathrm{c}}$ Depreciation
Depreciation tax shield $=.35(\$ 102,000)$
Depreciation tax shield $=\$ 35,700$
The depreciation tax shield shows us the increase in OCF by being able to expense depreciation.

## B-174 SOLUTIONS

5. The MACRS depreciation schedule is shown in Table 9.7. The ending book value for any year is the beginning book value minus the depreciation for the year. Remember, to find the amount of depreciation for any year, you multiply the purchase price of the asset times the MACRS percentage for the year. The depreciation schedule for this asset is:

| Beginning Year | Beginning <br> Book Value | Depreciati | Depreciation Allowance | Ending Book Value |
| :---: | :---: | :---: | :---: | :---: |
| 1 | \$860,000.00 | 14.29\% | \$122,894.00 | \$737,106.00 |
| 2 | 737,106.00 | 24.49\% | 210,614.00 | 526,492.00 |
| 3 | 526,492.00 | 17.49\% | 150,414.00 | 376,078.00 |
| 4 | 376,078.00 | 12.49\% | 107,414.00 | 268,664.00 |
| 5 | 268,664.00 | 8.93\% | 76,798.00 | 191,866.00 |
| 6 | 191,866.00 | 8.93\% | 76,798.00 | 115,068.00 |
| 7 | 115,068.00 | 8.93\% | 76,798.00 | 38,270.00 |
| 8 | 38,270.00 | 4.45\% | 38,270.00 | 0 |

6. The asset has an 8-year useful life and we want to find the BV of the asset after 5 years. With straightline depreciation, the depreciation each year will be:

Annual depreciation $=\$ 670,000 / 8$
Annual depreciation $=\$ 83,750$

So, after five years, the accumulated depreciation will be:
Accumulated depreciation $=5(\$ 83,750)$
Accumulated depreciation $=\$ 418,750$
The book value at the end of year five is thus:
$B V_{5}=\$ 670,000-418,750$
$B V_{5}=\$ 251,250$

The asset is sold at a loss to book value, so the depreciation tax shield of the loss is recaptured.
Aftertax salvage value $=\$ 95,000+(\$ 251,250-95,000)(0.35)$
Aftertax salvage value $=\$ 149,687.50$

To find the taxes on salvage value, remember to use the equation:
Taxes on salvage value $=(B V-M V) T_{c}$
This equation will always give the correct sign for a tax inflow (refund) or outflow (payment).
7. To find the BV at the end of four years, we need to find the accumulated depreciation for the first four years. We could calculate a table as in Problem 6, but an easier way is to add the MACRS depreciation amounts for each of the first four years and multiply this percentage times the cost of the asset. We can then subtract this from the asset cost. Doing so, we get:

$$
\begin{aligned}
& \mathrm{BV}_{4}=\$ 8,400,000-8,400,000(0.2000+0.3200+0.1920+0.1152) \\
& \mathrm{BV}_{4}=\$ 1,451,520
\end{aligned}
$$

The asset is sold at a gain to book value, so this gain is taxable.
Aftertax salvage value $=\$ 1,600,000+(\$ 1,451,520-1,600,000)(.34)$
Aftertax salvage value $=\$ 1,549,516.80$
8. We need to calculate the OCF, so we need an income statement. The lost sales of the current sound board sold by the company will be a negative since it will lose the sales.

| Sales of new | $\$ 43,200,000$ |
| :--- | ---: |
| Lost sales of old | $-3,375,000$ |
| Variable costs | $21,903,750$ |
| Fixed costs | $1,300,000$ |
| Depreciation | $1,500,000$ |
| EBT | $\$ 15,121,250$ |
| Tax | $\underline{5,746,075}$ |
| Net income | $\underline{\$ 9,375,175}$ |

The OCF for the company is:

$$
\begin{aligned}
& \mathrm{OCF}=\mathrm{EBIT}+\text { Depreciation }- \text { Taxes } \\
& \mathrm{OCF}=\$ 15,121,250+1,500,000-5,746,075 \\
& \mathrm{OCF}=\$ 10,875,175
\end{aligned}
$$

9. Using the tax shield approach to calculating OCF (Remember the approach is irrelevant; the final answer will be the same no matter which of the four methods you use.), we get:

OCF $=($ Sales - Costs $)\left(1-\mathrm{T}_{\mathrm{C}}\right)+\mathrm{T}_{\mathrm{C}}$ Depreciation
$\mathrm{OCF}=(\$ 2,450,000-1,180,000)(1-0.35)+0.35(\$ 2,700,000 / 3)$
$\mathrm{OCF}=\$ 1,140,500$
10. Since we have the OCF, we can find the NPV as the initial cash outlay, plus the PV of the OCFs, which are an annuity, so the NPV is:
$\mathrm{NPV}=-\$ 2,700,000+\$ 1,140,500\left(\mathrm{PVIFA}_{14 \%, 3}\right)$
$\mathrm{NPV}=-\$ 52,178.67$

## B-176 SOLUTIONS

11. The cash outflow at the beginning of the project will increase because of the spending on NWC. At the end of the project, the company will recover the NWC, so it will be a cash inflow. The sale of the equipment will result in a cash inflow, but we must also account for the taxes which will be paid on this sale. So, the cash flows for each year of the project will be:

| Year | Cash Flow |  |
| :---: | ---: | :--- |
| 0 | $-\$ 2,950,000$ | $=-\$ 2,700,000-250,000$ |
| 1 | $1,140,500$ |  |
| 2 | $1,140,500$ |  |
| 3 | $1,585,500$ | $=\$ 1,140,500+250,000+300,000+(0-300,000)(.35)$ |

And the NPV of the project is:

```
\(\mathrm{NPV}=-\$ 2,950,000+\$ 1,140,500\left(\mathrm{PVIFA}_{14 \%, 2}\right)+\left(\$ 1,585,500 / 1.14^{3}\right)\)
\(\mathrm{NPV}=-\$ 1,816.35\)
```

12. First, we will calculate the annual depreciation for the equipment necessary for the project. The depreciation amount each year will be:

Year 1 depreciation $=\$ 2,700,000(0.3333)=\$ 899,910$
Year 2 depreciation $=\$ 2,700,000(0.4444)=\$ 1,199,880$
Year 3 depreciation $=\$ 2,700,000(0.1482)=\$ 400,140$
So, the book value of the equipment at the end of three years, which will be the initial investment minus the accumulated depreciation, is:

Book value in 3 years $=\$ 2,700,000-(\$ 899,910+1,199,880+400,140)$
Book value in 3 years $=\$ 200,070$
The asset is sold at a gain to book value, so this gain is taxable.

$$
\text { Aftertax salvage value }=\$ 300,000+(\$ 200,070-300,000)(0.35)
$$

Aftertax salvage value $=\$ 265,025$
To calculate the OCF, we will use the tax shield approach, so the cash flow each year is:

$$
\begin{aligned}
& \text { OCF }=(\text { Sales }- \text { Costs })\left(1-\mathrm{T}_{\mathrm{C}}\right)+\mathrm{T}_{\mathrm{C}} \text { Depreciation } \\
& \begin{array}{cll}
\text { Year } & \underline{\text { Cash Flow }} & \\
1 & -\$ 2,950,000 & =-\$ 2,700,000-250,000 \\
2 & 1,140,468.50 & =(\$ 1,270,000)(.65)+0.35(\$ 899,910) \\
3 & 1,245,458.00 & =(\$ 1,270,000)(.65)+0.35(\$ 1,199,880) \\
1,480,573.50 & =(\$ 1,270,000)(.65)+0.35(\$ 400,140)+\$ 265,025+250,000
\end{array}
\end{aligned}
$$

Remember to include the NWC cost in Year 0, and the recovery of the NWC at the end of the project. The NPV of the project with these assumptions is:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 2,950,000+(\$ 1,140,468.50 / 1.14)+\left(\$ 1,245,458 / 1.14^{2}\right)+\left(\$ 1,480,573.50 / 1.14^{3}\right) \\
& \mathrm{NPV}=\$ 8,095.39
\end{aligned}
$$

13. First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation $=\$ 560,000 / 5$
Annual depreciation $=\$ 112,000$
Now, we calculate the aftertax salvage value. The aftertax salvage value is the market price minus (or plus) the taxes on the sale of the equipment, so:

Aftertax salvage value $=\mathrm{MV}+(\mathrm{BV}-\mathrm{MV}) \mathrm{T}_{\mathrm{C}}$
Very often, the book value of the equipment is zero, as it is in this case. If the book value is zero, the equation for the aftertax salvage value becomes:

Aftertax salvage value $=\mathrm{MV}+(0-\mathrm{MV}) \mathrm{T}_{\mathrm{C}}$
Aftertax salvage value $=M V\left(1-T_{C}\right)$
We will use this equation to find the aftertax salvage value since we know the book value is zero. So, the aftertax salvage value is:

Aftertax salvage value $=\$ 85,000(1-0.34)$
Aftertax salvage value $=\$ 56,100$
Using the tax shield approach, we find the OCF for the project is:
$\mathrm{OCF}=\$ 153,000(1-0.34)+0.34(\$ 112,000)$
OCF $=\$ 139,060$
Now we can find the project NPV. Notice we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

$$
\begin{aligned}
& \text { NPV }=-\$ 560,000-38,000+\$ 139,060\left(\text { PVIFA }_{8 \%, 5}\right)+\left[(\$ 56,100+38,000) / 1.08^{5}\right] \\
& \text { NPV }=\$ 21,269.14
\end{aligned}
$$

14. First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation charge $=\$ 780,000 / 5$
Annual depreciation charge $=\$ 156,000$
The aftertax salvage value of the equipment is:
Aftertax salvage value $=\$ 45,000(1-0.35)$
Aftertax salvage value $=\$ 29,250$

## B-178 SOLUTIONS

Using the tax shield approach, the OCF is:
$\mathrm{OCF}=\$ 310,000(1-0.35)+0.35(\$ 156,000)$
OCF $=\$ 256,100$

Now we can find the project IRR. There is an unusual feature that is a part of this project. Accepting this project means that we will reduce NWC. This reduction in NWC is a cash inflow at Year 0. This reduction in NWC implies that when the project ends, we will have to increase NWC. So, at the end of the project, we will have a cash outflow to restore the NWC to its level before the project. We must also include the aftertax salvage value at the end of the project. The IRR of the project is:
$\mathrm{NPV}=0=-\$ 780,000+55,000+\$ 256,100\left(\mathrm{PVIFA}_{\text {IRR } \%, 5}\right)+\left[(\$ 29,250-55,000) /(1+\mathrm{IRR})^{5}\right]$
IRR $=21.92 \%$
15. To evaluate the project with a $\$ 340,000$ cost savings, we need the OCF to compute the NPV. Using the tax shield approach, the OCF is:
$\mathrm{OCF}=\$ 340,000(1-0.35)+0.35(\$ 156,000)$
OCF $=\$ 275,600$
$\mathrm{NPV}=-\$ 780,000+55,000+\$ 275,600\left(\mathrm{PVIFA}_{20 \%, 5}\right)+\left[(\$ 29,250-55,000) /(1.20)^{5}\right]$
NPV = \$88,864.36

The NPV with a $\$ 280,000$ cost savings is:

```
\(\mathrm{OCF}=\$ 280,000(1-0.35)+0.35(\$ 156,000)\)
OCF \(=\$ 236,600\)
\(\mathrm{NPV}=-\$ 780,000+55,000+\$ 236,600\left(\mathrm{PVIFA}_{20 \%, 5}\right)+\left[(\$ 29,250-55,000) /(1.20)^{5}\right]\)
NPV \(=-\$ 27,769.52\)
```

We would accept the project if cost savings were $\$ 340,000$, and reject the project if the cost savings were $\$ 280,000$.
16. The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, sales and price increase, while costs decrease. In the worst case, sales and price decrease, and costs increase.

| Scenario | $\underline{\text { Unit Sales }}$ | $\underline{\text { Unit Price }}$ | Unit <br> Variable | $\underline{\underline{\text { Cost }}}$ |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Base case 80,000 $\$ 1,280$  $\$ 340$ | $\$ 5,500,000$ |  |  |  |  |
| Best case | 92,000 | $\$ 1,472$ |  | $\$ 289$ | $\$ 4,675,000$ |
| Worst case | 68,000 | $\$ 1,088$ | $\$ 391$ | $\$ 6,325,000$ |  |

17. An estimate for the impact of changes in price on the profitability of the project can be found from the sensitivity of NPV with respect to price; $\triangle \mathrm{NPV} / \Delta \mathrm{P}$. This measure can be calculated by finding the NPV at any two different price levels and forming the ratio of the changes in these parameters. Whenever a sensitivity analysis is performed, all other variables are held constant at their base-case values.
18. $a$. We will use the tax shield approach to calculate the OCF. The OCF is:
```
\(\mathrm{OCF}_{\text {base }}=[(\mathrm{P}-\mathrm{v}) \mathrm{Q}-\mathrm{FC}]\left(1-\mathrm{T}_{\mathrm{C}}\right)+\mathrm{T}_{\mathrm{C}} \mathrm{D}\)
OCF \(_{\text {base }}=[(\$ 36.50-22.75)(95,000)-\$ 830,000](0.65)+0.35(\$ 1,440,000 / 6)\)
OCF \(_{\text {base }}=\$ 393,562.50\)
```

Now we can calculate the NPV using our base-case projections. There is no salvage value or NWC, so the NPV is:

```
NPV base = $1,440,000 + $393,562.50(PVIFA 
NPV base = $133,285.69
```

To calculate the sensitivity of the NPV to changes in the quantity sold, we will calculate the NPV at a different quantity. We will use sales of 100,000 units. The NPV at this sales level is:

$$
\begin{aligned}
& \mathrm{OCF}_{\text {new }}=[(\$ 36.50-22.75)(100,000)-\$ 830,000](0.65)+0.35(\$ 1,440,000 / 6) \\
& \mathrm{OCF}_{\text {new }}=\$ 438,250
\end{aligned}
$$

And the NPV is:

```
\(\mathrm{NPV}_{\text {new }}=-\$ 1,440,000+\$ 438,250\left(\mathrm{PVIFA}_{13 \%, 6}\right)\)
\(\mathrm{NPV}_{\text {new }}=\$ 311,926.20\)
```

So, the change in NPV for every unit change in sales is:

```
\DeltaNPV/\DeltaS = [($133,285.69-311,926.50)]/(95,000 - 100,000)
\DeltaNPV/\DeltaS = +$35.728
```

If sales were to drop by 500 units, then NPV would drop by:
NPV drop $=\$ 35.728(500)$
NPV drop = \$17,864.05
You may wonder why we chose 100,000 units. Because it doesn't matter! Whatever sales number we use, when we calculate the change in NPV per unit sold, the ratio will be the same.

## B-180 SOLUTIONS

b. To find out how sensitive OCF is to a change in variable costs, we will compute the OCF at a variable cost of $\$ 21$. Again, the number we choose to use here is irrelevant: We will get the same ratio of OCF to a one dollar change in variable cost no matter what variable cost we use. So, using the tax shield approach, the OCF at a variable cost of $\$ 21$ is:

$$
\begin{aligned}
& \mathrm{OCF}_{\text {new }}=[(\$ 36.50-21)(95,000)-\$ 830,000](0.65)+0.35(\$ 1,440,000 / 6) \\
& \mathrm{OCF}_{\text {new }}=\$ 501,625.00
\end{aligned}
$$

So, the change in OCF for a $\$ 1$ change in variable costs is:

```
\DeltaOCF/\Deltav=($393,562.50-501,625.00)/($22.75-21)
\DeltaOCF/\Deltav = -$61,750
```

If variable costs decrease by $\$ 1$ then, OCF would increase by $\$ 61,750$.
19. We will use the tax shield approach to calculate the OCF for the best- and worst-case scenarios. For the best-case scenario, the price and quantity increase by 10 percent, so we will multiply the base case numbers by 1.1 , a 10 percent increase. The variable and fixed costs both decrease by 10 percent, so we will multiply the base case numbers by .9 , a 10 percent decrease. Doing so, we get:

$$
\begin{aligned}
\mathrm{OCF}_{\text {best }}= & \{[(\$ 36.50)(1.1)-(\$ 22.75)(0.9)](95,000)(1.1)-\$ 830,000(0.9)\}(0.65) \\
& +0.35(\$ 1,440,000 / 6) \\
\mathrm{OCF}_{\text {best }}= & \$ 934,874.38
\end{aligned}
$$

The best-case NPV is:

$$
\begin{aligned}
& \mathrm{NPV}_{\text {best }}=-\$ 1,440,000+\$ 934,874.38\left(\mathrm{PVIFA}_{13 \%, 6}\right) \\
& \mathrm{NPV}_{\text {best }}=\$ 2,297,206.86
\end{aligned}
$$

For the worst-case scenario, the price and quantity decrease by 10 percent, so we will multiply the base case numbers by .9 , a 10 percent decrease. The variable and fixed costs both increase by 10 percent, so we will multiply the base case numbers by 1.1 , a 10 percent increase. Doing so, we get:

$$
\begin{aligned}
& \mathrm{OCF}_{\text {worst }}=\{[(\$ 36.5)(0.9)-(\$ 22.75)(1.1)](95,000)(0.9)-\$ 830,000(1.1)\}(0.65)+0.35(\$ 1,440,000 / 6) \\
& \mathrm{OCF}_{\text {worst }}=-\$ 74,575.63
\end{aligned}
$$

The worst-case NPV is:

$$
\begin{aligned}
& \mathrm{NPV}_{\text {worst }}=-\$ 1,440,000-\$ 74,575.63\left(\mathrm{PVIFA}_{13 \%, 6}\right) \\
& \mathrm{NPV}_{\text {worst }}=-\$ 1,738,119.77
\end{aligned}
$$

20. First, we need to calculate the cash flows. The marketing study is a sunk cost and should be ignored. The net income each year will be:

| Sales of new product | $\$ 525,000$ |
| :--- | ---: |
| Variable costs | 105,000 |
| Fixed costs | 178,000 |
| Depreciation | $\underline{135,000}$ |
| EBT | $\$ 107,000$ |
| Tax | $\underline{42,800}$ |
| Net income | $\underline{\$ 64,200}$ |

So, the OCF is:
OCF $=$ EBIT + Depreciation - Taxes
OCF $=\$ 107,000+135,000-42,800$
OCF $=\$ 199,200$
The only initial cash flow is the cost of the equipment, so the payback period is:
Payback period $=\$ 540,000 / \$ 199,200$
Payback period $=2.71$ years
The NPV is:
NPV $=-\$ 540,000+\$ 199,200\left(\right.$ PVIFA $\left._{13 \%, 4}\right)$
$\mathrm{NPV}=\$ 52,514.69$
And the IRR is:
$0=-\$ 540,000+\$ 199,200\left(\right.$ PVIFA $\left._{\text {IRR }} \%, 4\right)$
IRR $=17.61 \%$

## Intermediate

21. First, we will calculate the depreciation each year, which will be:
$D_{1}=\$ 510,000(0.2000)=\$ 102,000$
$\mathrm{D}_{2}=\$ 510,000(0.3200)=\$ 163,200$
$\mathrm{D}_{3}=\$ 510,000(0.1920)=\$ 97,920$
$\mathrm{D}_{4}=\$ 510,000(0.1152)=\$ 58,752$
The book value of the equipment at the end of the project is:
$B V_{4}=\$ 510,000-(\$ 102,000+163,200+97,920+58,752)$
$\mathrm{BV}_{4}=\$ 88,128$

## B-182 SOLUTIONS

The asset is sold at a loss to book value, so this creates a tax refund.

```
After-tax salvage value = $64,000 + ($88,128-64,000)(0.34)
```

After-tax salvage value $=\$ 72,204$
Using the depreciation tax shield approach, the OCF for each year will be:
$\mathrm{OCF}_{1}=\$ 218,000(1-0.34)+0.34(\$ 102,000)=\$ 178,560$
$\mathrm{OCF}_{2}=\$ 218,000(1-0.34)+0.34(\$ 163,200)=\$ 199,368$
$\mathrm{OCF}_{3}=\$ 218,000(1-0.34)+0.34(\$ 97,920)=\$ 177,173$
$\mathrm{OCF}_{4}=\$ 218,000(1-0.34)+0.34(\$ 58,752)=\$ 163,856$
Now, we have all the necessary information to calculate the project NPV. We need to be careful with the NWC in this project. Notice the project requires $\$ 21,000$ of NWC at the beginning, and $\$ 3,000$ more in NWC each successive year. We will subtract the $\$ 21,000$ from the initial cash flow, and subtract $\$ 3,000$ each year from the OCF to account for this spending. In Year 4, we will add back the total spent on NWC, which is $\$ 30,000$. The $\$ 3,000$ spent on NWC capital during Year 4 is irrelevant. Why? Well, during this year the project required an additional $\$ 3,000$, but we would get the money back immediately. So, the net cash flow for additional NWC would be zero. With all this, the equation for the NPV of the project is:
$\mathrm{NPV}=-\$ 510,000-21,000+(\$ 178,560-3,000) / 1.11+(\$ 199,368-3,000) / 1.11^{2}$

$$
+(\$ 177,173-3,000) / 1.11^{3}+(\$ 163,856+30,000+72,204) / 1.11^{4}
$$

NPV $=\$ 89,153.92$
22. Using the tax shield approach, the OCF at 100,000 units will be:
$\mathrm{OCF}=[(\mathrm{P}-\mathrm{v}) \mathrm{Q}-\mathrm{FC}]\left(1-\mathrm{T}_{\mathrm{C}}\right)+\mathrm{T}_{\mathrm{C}}(\mathrm{D})$
OCF $=[(\$ 31.75-19.50)(80,000)-185,000](0.66)+0.34(\$ 730,000 / 5)$
OCF $=\$ 574,340$
We will calculate the OCF at 81,000 units. The choice of the second level of quantity sold is arbitrary and irrelevant. No matter what level of units sold we choose, we will still get the same sensitivity. So, the OCF at this level of sales is:

OCF $=[(\$ 31.75-19.50)(81,000)-185,000](0.66)+0.34(\$ 730,000 / 5)$
OCF $=\$ 582,425$
The sensitivity of the OCF to changes in the quantity sold is:

$$
\begin{aligned}
& \text { Sensitivity }=\Delta \mathrm{OCF} / \Delta \mathrm{Q}=(\$ 574,340-582,425) /(80,000-81,000) \\
& \Delta \mathrm{OCF} / \Delta \mathrm{Q}=+\$ 8.09
\end{aligned}
$$

OCF will increase by $\$ 8.09$ for every additional unit sold.
23. $a$. The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, sales and price increase, while costs decrease. In the worst case, sales and price decrease, and costs increase.

| Scenario | Unit sales |  | Variable cost |  |
| :--- | ---: | ---: | ---: | ---: |
| Base | 190 | $\$ 14,600$ |  | $\$ 295,000$ |
| Best | 209 | $\$ 13,140$ | $\$ 265,500$ |  |
| Worst | 171 | $\$ 16,060$ | $\$ 324,500$ |  |

Using the tax shield approach, the OCF and NPV for the base case estimate is:

$$
\begin{aligned}
& \mathrm{OCF}_{\text {base }}=[(\$ 18,200-14,600)(190)-\$ 295,000](0.65)+0.35(\$ 890,000 / 4) \\
& \mathrm{OCF}_{\text {base }}=\$ 330,725 \\
& \mathrm{NPV}_{\text {base }}=-\$ 890,000+\$ 330,725\left(\mathrm{PVIFA}_{12 \%, 4}\right) \\
& \mathrm{NPV}_{\text {base }}=\$ 114,527.36
\end{aligned}
$$

The OCF and NPV for the worst case estimate are:

```
\(\mathrm{OCF}_{\text {worst }}=[(\$ 18,200-16,060)(171)-\$ 324,500](0.65)+0.35(\$ 890,000 / 4)\)
\(\mathrm{OCF}_{\text {worst }}=\$ 104,811\)
\(\mathrm{NPV}_{\text {worst }}=-\$ 890,000+\$ 104,811\left(\mathrm{PVIFA}_{12 \%, 4}\right)\)
\(\mathrm{NPV}_{\text {worst }}=-\$ 571,652.38\)
```

And the OCF and NPV for the best case estimate are:
$\mathrm{OCF}_{\text {best }}=[(\$ 18,200-13,140)(209)-\$ 265,500](0.65)+0.35(\$ 890,000 / 4)$
$\mathrm{OCF}_{\text {best }}=\$ 592,701$
$\mathrm{NPV}_{\text {best }}=-\$ 890,000+\$ 592,701\left(\mathrm{PVIFA}_{12 \%, 4}\right)$
$\mathrm{NPV}_{\text {best }}=\$ 910,240.00$
b. To calculate the sensitivity of the NPV to changes in fixed costs, we choose another level of fixed costs. We will use fixed costs of $\$ 300,000$. The OCF using this level of fixed costs and the other base-case values with the tax shield approach, we get:
$\mathrm{OCF}=[(\$ 18,200-14,600)(190)-\$ 300,000](0.65)+0.35(\$ 890,000 / 4)$
$\mathrm{OCF}=\$ 327,475$

And the NPV is:
$\mathrm{NPV}=-\$ 890,000+\$ 327,475\left(\mathrm{PVIFA}_{12 \%, 4}\right)$
$\mathrm{NPV}=\$ 104,655.98$

## B-184 SOLUTIONS

The sensitivity of NPV to changes in fixed costs is:

```
\DeltaNPV/\DeltaFC = ($114,527.36-104,655.98)/($295,000 - 300,000)
\DeltaNPV/\DeltaFC = -$1.974
```

For every dollar FC increase, NPV falls by $\$ 1.97$.
24. The marketing study and the research and development are both sunk costs and should be ignored. The initial cost is the equipment plus the net working capital, so:

Initial cost $=\$ 19,800,000+1,500,000$
Initial cost $=\$ 21,300,000$
Next, we will calculate the sales and variable costs. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

| Sales |  |
| :--- | ---: |
| New clubs | $\$ 675 \times 70,000=\$ 47,250,000$ |
| Exp. clubs | $\$ 1,100 \times(-9,000)=-9,900,000$ |
| Cheap clubs | $\$ 300 \times 12,000=\frac{3,600,000}{\$ 40,950,000}$ |

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets anymore, we will save these variable costs, which is an inflow. So:

| Var. costs |  |
| :--- | ---: |
| New clubs | $\$ 340 \times 70,000=\$ 23,800,000$ |
| Exp. clubs | $\$ 550 \times(-9,000)=-4,950,000$ |
| Cheap clubs | $\$ 100 \times 12,000=\frac{1,200,000}{}$ |
|  |  |

The pro forma income statement will be:

| Sales | $\$ 40,950,000$ |
| :--- | ---: |
| Variable costs | $20,050,000$ |
| Costs | $10,800,000$ |
| Depreciation | $\underline{2,828,571}$ |
| EBT | $7,271,429$ |
| Taxes | $\underline{2,908,571}$ |
| Net income | $\underline{\$ 4,362,857}$ |

Using the bottom up OCF calculation, we get:
$\mathrm{OCF}=\mathrm{NI}+$ Depreciation
$\mathrm{OCF}=\$ 4,362,857+2,828,571$
OCF $=\$ 7,191,429$
So, the payback period is:
Payback period $=2+\$ 6,917,142 / \$ 7,191,429$
Payback period $=2.96$ years
The NPV is:

$$
\begin{aligned}
& \text { NPV }=-\$ 21,300,000-1,500,000+\$ 7,191,429\left(\mathrm{PVIFA}_{14 \%, 7}\right)+\$ 1,500,000 / 1.14^{7} \\
& \mathrm{NPV}=\$ 10,138,493.93
\end{aligned}
$$

And the IRR is:
$0=-\$ 21,300,000-1,500,000+\$ 7,191,429\left(\mathrm{PVIFA}_{\text {IRR }}, 7\right)+\$ 1,500,000 / \mathrm{IRR}^{7}$
$\operatorname{IRR}=28.17 \%$

## Challenge

25. This is an in-depth capital budgeting problem. Probably the easiest OCF calculation for this problem is the bottom up approach, so we will construct an income statement for each year. Beginning with the initial cash flow at time zero, the project will require an investment in equipment. The project will also require an investment in NWC. The NWC investment will be 15 percent of the next year's sales. In this case, it will be Year 1 sales. Realizing we need Year 1 sales to calculate the required NWC capital at time 0 , we find that Year 1 sales will be $\$ 29,920,000$. So, the cash flow required for the project today will be:

| Capital spending | $\$ 22,000,000$ <br> Change in NWC <br> Total cash flow |
| :--- | ---: |
| $-1,500,000$ |  |
| $\$ 23,500,000$ |  |

Now we can begin the remaining calculations. Sales figures are given for each year, along with the price per unit. The variable costs per unit are used to calculate total variable costs, and fixed costs are given at $\$ 850,000$ per year. To calculate depreciation each year, we use the initial equipment cost of $\$ 22$ million, times the appropriate MACRS depreciation each year. The remainder of each income statement is calculated below. Notice at the bottom of the income statement we added back depreciation to get the OCF for each year. The section labeled "Net cash flows" will be discussed below:

| Year | $\underline{1}$ | $\underline{2}$ | $\underline{3}$ |  | $\underline{5}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Ending book value | $\$ 18,856,200$ | $\$ 13,468,400$ | $\$ 9,620,600$ | $\$ 6,872,800$ | $\$ 4,908,200$ |
|  |  |  |  |  |  |
| Sales | $\$ 29,920,000$ | $\$ 32,640,000$ | $\$ 37,060,000$ | $\$ 40,120,000$ | $\$ 32,300,000$ |
| Variable costs | $21,120,000$ | $23,040,000$ | $26,160,000$ | $28,320,000$ | $22,800,000$ |
| Fixed costs | 850,000 | 850,000 | 850,000 | 850,000 | 850,000 |
| Depreciation | $3,143,800$ | $5,387,800$ | $3,847,800$ | $2,747,800$ | $1,964,600$ |
| EBIT | $4,806,200$ | $3,362,200$ | $6,202,200$ | $8,202,200$ | $6,685,400$ |
| Taxes | $1,682,170$ | $1,176,770$ | $2,170,770$ | $2,870,770$ | $2,339,890$ |
| Net income | $3,124,030$ | $2,185,430$ | $4,031,430$ | $5,331,430$ | $4,345,510$ |
| Depreciation | $3,143,800$ | $5,387,800$ | $3,847,800$ | $2,747,800$ | $1,964,600$ |
| Operating cash flow | $\$ 6,267,830$ | $\$ 7,573,230$ | $\$ 7,879,230$ | $\$ 8,079,230$ | $\$ 6,310,110$ |

## Net cash flows

| Operating cash flow | $\$ 6,267,830$ | $\$ 7,573,230$ | $\$ 7,879,230$ | $\$ 8,079,230$ | $\$ 6,310,110$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Change in NWC | $-408,000$ | $-663,000$ | $-459,000$ | $1,173,000$ | $1,857,000$ |
| Capital spending | 0 | 0 | 0 | 0 | $4,577,870$ |
| Total cash flow | $\$ 5,859,830$ | $\$ 6,910,230$ | $\$ 7,420,230$ | $\$ 9,252,230$ | $\$ 12,744,980$ |

After we calculate the OCF for each year, we need to account for any other cash flows. The other cash flows in this case are NWC cash flows and capital spending, which is the aftertax salvage of the equipment. The required NWC capital is 15 percent of the sales in the next year. We will work through the NWC cash flow for Year 1. The total NWC in Year 1 will be 15 percent of sales increase from Year 1 to Year 2, or:

Increase in NWC for Year $1=.15(\$ 32,640,000-29,920,000)$
Increase in NWC for Year $1=\$ 408,000$
Notice that the NWC cash flow is negative. Since the sales are increasing, we will have to spend more money to increase NWC. In Year 4, the NWC cash flow is positive since sales are declining. And, in Year 5, the NWC cash flow is the recovery of all NWC the company still has in the project.

To calculate the aftertax salvage value, we first need the book value of the equipment. The book value at the end of the five years will be the purchase price, minus the total depreciation. So, the ending book value is:

Ending book value $=\$ 22,000,000-(\$ 3,143,800+5,387,800+3,847,800+2,747,800$

$$
+1,964,600)
$$

Ending book value $=\$ 4,908,200$
The market value of the used equipment is 20 percent of the purchase price, or $\$ 4.4$ million, so the aftertax salvage value will be:

Aftertax salvage value $=\$ 4,400,000+(\$ 4,908,200-4,400,000)(.35)$
Aftertax salvage value $=\$ 4,577,870$
The aftertax salvage value is included in the total cash flows are capital spending. Now we have all of the cash flows for the project. The NPV of the project is:
$\mathrm{NPV}=-\$ 23,500,000+\$ 5,859,830 / 1.18+\$ 6,910,230 / 1.18^{2}+\$ 7,420,230 / 1.18^{3}$
$+\$ 9,252,230 / 1.18^{4}+\$ 12,744,980 / 1.18^{5}$
$\mathrm{NPV}=\$ 1,288,103.80$

And the IRR is:
$\mathrm{NPV}=0=-\$ 23,500,000+\$ 5,859,830 /(1+\mathrm{IRR})+\$ 6,910,230 /(1+\mathrm{IRR})^{2}$ $+\$ 7,420,230 /(1+\text { IRR })^{3}+\$ 9,252,230 /(1+\operatorname{IRR})^{4}+\$ 12,744,980 /(1+\text { IRR })^{5}$
$\operatorname{IRR}=20.10 \%$
We should accept the project.
26. To find the initial pretax cost savings necessary to buy the new machine, we should use the tax shield approach to find the OCF. We begin by calculating the depreciation each year using the MACRS depreciation schedule. The depreciation each year is:
$\mathrm{D}_{1}=\$ 540,000(0.3333)=\$ 179,982$
$\mathrm{D}_{2}=\$ 540,000(0.4444)=\$ 239,976$
$\mathrm{D}_{3}=\$ 540,000(0.1482)=\$ 80,028$
$\mathrm{D}_{4}=\$ 540,000(0.0741)=\$ 40,014$
Using the tax shield approach, the OCF each year is:

$$
\begin{aligned}
& \mathrm{OCF}_{1}=(\mathrm{S}-\mathrm{C})(1-0.35)+0.35(\$ 179,982) \\
& \mathrm{OCF}_{2}=(\mathrm{S}-\mathrm{C})(1-0.35)+0.35(\$ 239,976) \\
& \mathrm{OCF}_{3}=(\mathrm{S}-\mathrm{C})(1-0.35)+0.35(\$ 80,028) \\
& \mathrm{OCF}_{4}=(\mathrm{S}-\mathrm{C})(1-0.35)+0.35(\$ 40,014) \\
& \mathrm{OCF}_{5}=(\mathrm{S}-\mathrm{C})(1-0.35)
\end{aligned}
$$

Now we need the aftertax salvage value of the equipment. The aftertax salvage value is:
After-tax salvage value $=\$ 60,000(1-0.35)=\$ 39,000$

## B-188 SOLUTIONS

To find the necessary cost reduction, we must realize that we can split the cash flows each year. The OCF in any given year is the cost reduction ( $\mathrm{S}-\mathrm{C}$ ) times one minus the tax rate, which is an annuity for the project life, and the depreciation tax shield. To calculate the necessary cost reduction, we would require a zero NPV. The equation for the NPV of the project is:

$$
\begin{aligned}
\text { NPV }=0 & =-\$ 540,000-40,000+(\mathrm{S}-\mathrm{C})(0.65)\left(\mathrm{PVIFA}_{12 \%, 5}\right)+0.35(\$ 179,982 / 1.12 \\
& \left.+\$ 239,976 / 1.12^{2}+\$ 80,028 / 1.12^{3}+\$ 40,014 / 1.12^{4}\right)+(\$ 40,000+39,000) / 1.12^{5}
\end{aligned}
$$

Solving this equation for the sales minus costs, we get:
$(\mathrm{S}-\mathrm{C})(0.65)\left(\mathrm{PVIFA}_{12 \%, 5}\right)=\$ 383,134.12$
$(S-C)=\$ 163,515.59$

## CHAPTER 10 SOME LESSONS FROM CAPITAL MARKET HISTORY

## Answers to Concepts Review and Critical Thinking Questions

1. They all wish they had! Since they didn't, it must have been the case that the stellar performance was not foreseeable, at least not by most.
2. As in the previous question, it's easy to see after the fact that the investment was terrible, but it probably wasn't so easy ahead of time.
3. No, stocks are riskier. Some investors are highly risk averse, and the extra possible return doesn't attract them relative to the extra risk.
4. On average, the only return that is earned is the required return-investors buy assets with returns in excess of the required return (positive NPV), bidding up the price and thus causing the return to fall to the required return (zero NPV); investors sell assets with returns less than the required return (negative NPV), driving the price lower and thus the causing the return to rise to the required return (zero NPV).
5. The market is not weak form efficient.
6. Yes, historical information is also public information; weak form efficiency is a subset of semi-strong form efficiency.
7. Ignoring trading costs, on average, such investors merely earn what the market offers; the trades all have zero NPV. If trading costs exist, then these investors lose by the amount of the costs.
8. Unlike gambling, the stock market is a positive sum game; everybody can win. Also, speculators provide liquidity to markets and thus help to promote efficiency.
9. The EMH only says, that within the bounds of increasingly strong assumptions about the information processing of investors, that assets are fairly priced. An implication of this is that, on average, the typical market participant cannot earn excessive profits from a particular trading strategy. However, that does not mean that a few particular investors cannot outperform the market over a particular investment horizon. Certain investors who do well for a period of time get a lot of attention from the financial press, but the scores of investors who do not do well over the same period of time generally get considerably less attention.
10. a. If the market is not weak form efficient, then this information could be acted on and a profit earned from following the price trend. Under (2), (3), and (4), this information is fully impounded in the current price and no abnormal profit opportunity exists.

## B-190 SOLUTIONS

b. Under (2), if the market is not semi-strong form efficient, then this information could be used to buy the stock "cheap" before the rest of the market discovers the financial statement anomaly. Since (2) is stronger than (1), both imply that a profit opportunity exists; under (3) and (4), this information is fully impounded in the current price and no profit opportunity exists.
c. Under (3), if the market is not strong form efficient, then this information could be used as a profitable trading strategy, by noting the buying activity of the insiders as a signal that the stock is underpriced or that good news is imminent. Since (1) and (2) are weaker than (3), all three imply that a profit opportunity exists. Note that this assumes the individual who sees the insider trading is the only one who sees the trading. If the information about the trades made by company management is public information, under (3) it will be discounted in the stock price and no profit opportunity exists. Under (4), this information does not signal any profit opportunity for traders; any pertinent information the manager-insiders may have is fully reflected in the current share price.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. The return of this stock is:
$\mathrm{R}=[(\$ 87-78)+1.25] / \$ 78$
$\mathrm{R}=.1314$ or $13.14 \%$
2. The dividend yield is the dividend divided by price at the beginning of the period price, so:

Dividend yield $=\$ 1.25 / \$ 78$
Dividend yield $=.0160$ or $1.60 \%$
And the capital gains yield is the increase in price divided by the initial price, so:
Capital gains yield $=(\$ 87-78) / \$ 78$
Capital gains yield $=.1154$ or $11.54 \%$
3. Using the equation for total return, we find:
$\mathrm{R}=[(\$ 71-78)+1.25] / \$ 78$
$\mathrm{R}=-.0737$ or $-7.37 \%$
And the dividend yield and capital gains yield are:
Dividend yield $=\$ 1.25 / \$ 78$
Dividend yield $=.0160$ or $1.60 \%$

Capital gains yield $=(\$ 71-78) / \$ 78$
Capital gains yield $=-.0897$ or $-8.97 \%$
Here's a question for you: Can the dividend yield ever be negative? No, that would mean you were paying the company for the privilege of owning the stock, however, it has happened on bonds.
4. The total dollar return is the change in price plus the coupon payment, so:

Total dollar return $=\$ 1,063-1,090+80$
Total dollar return $=\$ 53$
The total percentage return of the bond is:
$\mathrm{R}=[(\$ 1,063-1,090)+80] / \$ 1,090$
$\mathrm{R}=.0486$ or $4.86 \%$
Notice here that we could have simply used the total dollar return of $\$ 53$ in the numerator of this equation.

Using the Fisher equation, the real return was:
$(1+\mathrm{R})=(1+\mathrm{r})(1+\mathrm{h})$
$\mathrm{r}=(1.0486 / 1.03)-1$
$r=.0181$ or $1.81 \%$
5. The nominal return is the stated return, which is 12.30 percent. Using the Fisher equation, the real return was:
$(1+\mathrm{R})=(1+\mathrm{r})(1+\mathrm{h})$
$\mathrm{r}=(1.1230) /(1.031)-1$
$\mathrm{r}=.0892$ or $8.92 \%$
6. Using the Fisher equation, the real returns for government and corporate bonds were:

$$
\begin{aligned}
& (1+\mathrm{R})=(1+\mathrm{r})(1+\mathrm{h}) \\
& \mathrm{r}_{\mathrm{G}}=1.058 / 1.031-1 \\
& \mathrm{r}_{\mathrm{G}}=.0262 \text { or } 2.62 \% \\
& \mathrm{r}_{\mathrm{C}}=1.062 / 1.031-1 \\
& \mathrm{r}_{\mathrm{C}}=.0301 \text { or } 3.01 \%
\end{aligned}
$$

## B-192 SOLUTIONS

7. The average return is the sum of the returns, divided by the number of returns. The average return for each stock was:

$$
\begin{aligned}
& \bar{X}=\left[\sum_{i=1}^{N} x_{i}\right] / N=\frac{[.15+.04-.13+.34+.17]}{5}=.1140 \text { or } 11.40 \% \\
& \bar{Y}=\left[\sum_{i=1}^{N} y_{i}\right] / N=\frac{[.29-.06+.19-.07+.38]}{5}=.1460 \text { or } 14.60 \%
\end{aligned}
$$

Remembering back to "sadistics," we calculate the variance of each stock as:

$$
\begin{aligned}
& s_{X}{ }^{2}=\left[\sum_{i=1}^{N}\left(x_{i}-\bar{x}\right)^{2}\right] /(N-1) \\
& s_{X}{ }^{2}=\frac{1}{5-1}\left\{(.15-.114)^{2}+(.04-.114)^{2}+(-.13-.114)^{2}+(.34-.114)^{2}+(.17-.114)^{2}\right\}=.030130 \\
& s_{Y}{ }^{2}=\frac{1}{5-1}\left\{(.29-.146)^{2}+(-.06-.146)^{2}+(.19-.146)^{2}+(-.07-.146)^{2}+(.38-.146)^{2}\right\}=.041630
\end{aligned}
$$

The standard deviation is the square root of the variance, so the standard deviation of each stock is:
$s_{X}=(.030130)^{1 / 2}$
$s_{X}=.1736$ or $17.36 \%$
$s_{Y}=(.041630)^{1 / 2}$
$s_{Y}=.2040$ or $20.40 \%$
8. We will calculate the sum of the returns for each asset and the observed risk premium first. Doing so, we get:

| Year | Large co. stock return | T-bill return | Risk premium |
| :---: | :---: | :---: | :---: |
| 1973 | -14.69\% | 7.29\% | -21.98\% |
| 1974 | -26.47 | 7.99 | -34.46 |
| 1975 | 37.23 | 5.87 | 31.36 |
| 1976 | 23.93 | 5.07 | 18.86 |
| 1977 | -7.16 | 5.45 | -12.61 |
| 1978 | 6.57 | 7.64 | -1.07 |
|  | 19.41 | 39.31 | -19.90 |

a. The average return for large company stocks over this period was:

Large company stock average return $=19.41 \% / 6$
Large company stock average return $=3.24 \%$

And the average return for T-bills over this period was:
T-bills average return $=39.31 \% / 6$
T -bills average return $=6.55 \%$
b. Using the equation for variance, we find the variance for large company stocks over this period was:

Variance $=1 / 5\left[(-.1469-.0324)^{2}+(-.2647-.0324)^{2}+(.3723-.0324)^{2}+(.2393-.0324)^{2}+\right.$ $\left.(-.0716-.0324)^{2}+(.0657-.0324)^{2}\right]$
Variance $=0.058136$
And the standard deviation for large company stocks over this period was:
Standard deviation $=(0.058136)^{1 / 2}$
Standard deviation $=0.2411$ or $24.11 \%$
Using the equation for variance, we find the variance for T-bills over this period was:
Variance $=1 / 5\left[(.0729-.0655)^{2}+(.0799-.0655)^{2}+(.0587-.0655)^{2}+(.0507-.0655)^{2}+\right.$ $\left.(.0545-.0655)^{2}+(.0764-.0655)^{2}\right]$
Variance $=0.000153$
And the standard deviation for T-bills over this period was:
Standard deviation $=(0.000153)^{1 / 2}$
Standard deviation $=0.0124$ or $1.24 \%$
c. The average observed risk premium over this period was:

Average observed risk premium $=-19.90 \% / 6$
Average observed risk premium $=-3.32 \%$
The variance of the observed risk premium was:
Variance $=1 / 5\left[(-.2198-.0332)^{2}+(-.3446-.0332)^{2}+(.3136-.0332)^{2}+\right.$
$\left.(.1886-.0332)^{2}+(-.1261-.0332)^{2}+(-.0107-.0332)^{2}\right]$
Variance $=0.062078$
And the standard deviation of the observed risk premium was:
Standard deviation $=(0.062078)^{1 / 2}$
Standard deviation $=0.2492$ or $24.92 \%$
d. Before the fact, for most assets the risk premium will be positive; investors demand compensation over and above the risk-free return to invest their money in the risky asset. After the fact, the observed risk premium can be negative if the asset's nominal return is unexpectedly low, the riskfree return is unexpectedly high, or if some combination of these two events occurs.

## B-194 SOLUTIONS

9. $a$. To find the average return, we sum all the returns and divide by the number of returns, so:

Arithmetic average return $=(-.25+.36+.09+.11+.17) / 5$
Arithmetic average return $=.0960$ or $9.60 \%$
b. Using the equation to calculate variance, we find:

Variance $=1 / 4\left[(-.25-.096)^{2}+(.36-.096)^{2}+(.09-.096)^{2}+(.11-.096)^{2}+\right.$ $\left.(.17-.096)^{2}\right]$
Variance $=0.048780$
So, the standard deviation is:
Standard deviation $=(0.048780)^{1 / 2}$
Standard deviation $=0.2209$ or $22.09 \%$
10. a. To calculate the average real return, we can use the average return of the asset, and the average risk-free rate in the Fisher equation. Doing so, we find:
$(1+\mathrm{R})=(1+\mathrm{r})(1+\mathrm{h})$
$\overline{\mathrm{r}}=(1.0960 / 1.042)-1$
$\overline{\mathrm{r}}=.0518$ or $5.18 \%$
b. The average risk premium is simply the average return of the asset, minus the average risk-free rate, so, the average risk premium for this asset would be:

$$
\begin{aligned}
& \overline{\mathrm{RP}}=\overline{\mathrm{R}}-\overline{\mathrm{R}}_{\mathrm{f}} \\
& \overline{\mathrm{RP}}=.0960-.0510 \\
& \overline{\mathrm{RP}}=.0450 \text { or } 4.50 \%
\end{aligned}
$$

11. We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

$$
\begin{aligned}
& (1+\mathrm{R})=(1+\mathrm{r})(1+\mathrm{h}) \\
& \overline{\mathrm{r}}_{\mathrm{f}}=(1.051 / 1.042)-1 \\
& \overline{\mathrm{r}}_{\mathrm{f}}=.0086 \text { or } 0.86 \%
\end{aligned}
$$

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:
$\overline{\mathrm{rp}}=\overline{\mathrm{r}}-\overline{\mathrm{r}}_{\mathrm{f}}=5.18 \%-0.86$
$\overline{\mathrm{rp}}=.0432$ or $4.32 \%$
12. T-bill rates were highest in the early eighties. This was during a period of high inflation and is consistent with the Fisher effect.
13. To find the return on the zero coupon bond, we first need to find the price of the bond today. We need to remember that the price for zero coupon bonds is calculated with semiannual periods. Since one year has elapsed, the bond now has 19 years to maturity, so the price today is:
$\mathrm{P}_{1}=\$ 1,000 / 1.045^{38}$
$\mathrm{P}_{1}=\$ 187.75$
There are no intermediate cash flows on a zero coupon bond, so the return is the capital gains, or:
$\mathrm{R}=(\$ 187.75-162.87) / \$ 162.87$
$R=.1528$ or $15.28 \%$
14. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. Since preferred stock is assumed to have a par value of $\$ 100$, the dividend was $\$ 6$, so the return for the year was:
$\mathrm{R}=(\$ 93.80-95.12+6.00) / \$ 95.12$
$\mathrm{R}=.0492$ or $4.92 \%$
15. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This stock paid no dividend, so the return was:

$$
\begin{aligned}
& \mathrm{R}=(\$ 46.81-41.05) / \$ 41.05 \\
& \mathrm{R}=.1403 \text { or } 14.03 \%
\end{aligned}
$$

This is the return for three months, so the APR is:

$$
\begin{aligned}
& \mathrm{APR}=4(14.03 \%) \\
& \mathrm{APR}=56.13 \%
\end{aligned}
$$

And the EAR is:

$$
\begin{aligned}
& \mathrm{EAR}=(1+.1403)^{4}-1 \\
& \mathrm{EAR}=.6908 \text { or } 69.08 \%
\end{aligned}
$$

## B-196 SOLUTIONS

16. To find the real return each year, we will use the Fisher equation, which is:
$1+\mathrm{R}=(1+\mathrm{r})(1+\mathrm{h})$
Using this relationship for each year, we find:

|  | T-bills |  | Inflation |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Real Return |  |  |  |
| 1926 | 0.0330 |  | $(0.0112)$ | 0.0447 |
| 1927 | 0.0315 |  | $(0.0226)$ | 0.0554 |
| 1928 | 0.0405 |  | $(0.0116)$ | 0.0527 |
| 1929 | 0.0447 | 0.0058 | 0.0387 |  |
| 1930 | 0.0227 | $(0.0640)$ | 0.0926 |  |
| 1931 | 0.0115 | $(0.0932)$ | 0.1155 |  |
| 1932 | 0.0088 | $(0.1027)$ | 0.1243 |  |

So, the average real return was:
Average $=(.0447+.0554+.0527+.0387+.0926+.1155+.1243) / 7$
Average $=.0748$ or $7.48 \%$
Notice that the real return was higher than the nominal return during this period because of deflation, or negative inflation.
17. Looking at the long-term corporate bond return history in Figure 10.10, we see that the mean return was 6.2 percent, with a standard deviation of 8.5 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:
$R \in \mu \pm 1 \sigma=6.2 \% \pm 8.5 \%=-2.30 \%$ to $14.70 \%$
The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:
$R \in \mu \pm 2 \sigma=6.2 \% \pm 2(8.5 \%)=-10.80 \%$ to $23.20 \%$
18. Looking at the large-company stock return history in Figure 10.10, we see that the mean return was 12.3 percent, with a standard deviation of 20.1 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:
$R \in \mu \pm 1 \sigma=12.3 \% \pm 20.1 \%=-7.80 \%$ to $32.40 \%$
The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:
$R \in \mu \pm 2 \sigma=12.3 \% \pm 2(20.1 \%)=-27.90 \%$ to $52.50 \%$

## Intermediate

19. Here, we know the average stock return, and four of the five returns used to compute the average return. We can work the average return equation backward to find the missing return. The average return is calculated as:

$$
\begin{aligned}
& .11=(.13-.18+.09+.36+\mathrm{R}) / 5 \\
& .55=.13-.18+.09+.36+\mathrm{R} \\
& \mathrm{R}=.15 \text { or } 15 \%
\end{aligned}
$$

The missing return has to be 15 percent. Now, we can use the equation for the variance to find:
Variance $=1 / 4\left[(.13-.11)^{2}+(-.18-.11)^{2}+(.09-.11)^{2}+(.36-.11)^{2}+(.15-.11)^{2}\right]$
Variance $=0.037250$
And the standard deviation is:
Standard deviation $=(0.037250)^{1 / 2}$
Standard deviation $=0.1930$ or $19.30 \%$
20. The arithmetic average return is the sum of the known returns divided by the number of returns, so:

Arithmetic average return $=(.36+.19+.27-.07+.06-.13) / 6$
Arithmetic average return $=.1567$ or $15.67 \%$
Using the equation for the geometric return, we find:
Geometric average return $=\left[\left(1+\mathrm{R}_{1}\right) \times\left(1+\mathrm{R}_{2}\right) \times \ldots \times\left(1+\mathrm{R}_{T}\right)\right]^{1 / T}-1$
Geometric average return $=[(1+.36)(1+.19)(1+.27)(1-.07)(1+.06)(1-.13)]^{(1 / 6)}-1$
Geometric average return $=.1480$ or $14.80 \%$
Remember that the geometric average return will always be less than the arithmetic average return if the returns have any variation.
21. To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:
$\mathrm{R}_{1}=(\$ 53.84-48.27+0.57) / \$ 48.27=.1272$ or $12.72 \%$
$\mathrm{R}_{2}=(\$ 57.75-53.84+0.62) / \$ 53.84=.0841$ or $8.41 \%$
$\mathrm{R}_{3}=(\$ 54.21-57.75+0.68) / \$ 57.75=-.0495$ or $-4.95 \%$
$\mathrm{R}_{4}=(\$ 62.09-54.21+0.77) / \$ 54.21=.1596$ or $15.96 \%$
$\mathrm{R}_{5}=(\$ 71.83-62.09+0.84) / \$ 62.09=.1704$ or $17.04 \%$
The arithmetic average return was:
$\mathrm{R}_{\mathrm{A}}=(0.1272+0.0841-0.0495+0.1596+0.1704) / 5$
$\mathrm{R}_{\mathrm{A}}=0.0984$ or $9.84 \%$

## B-198 SOLUTIONS

And the geometric average return was:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{G}}=[(1+.1272)(1+.0841)(1-.0495)(1+.1596)(1+.1704)]^{1 / 5}-1 \\
& \mathrm{R}_{\mathrm{G}}=0.0953 \text { or } 9.53 \%
\end{aligned}
$$

22. To find the real return, we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:
$\mathrm{r}=[(1+\mathrm{R}) /(1+\mathrm{h})]-1$
So, the real return each year was:

| Year | T-bill return | Inflation | Real return |
| :---: | :---: | :---: | :---: |
| 1973 | 0.0729 | 0.0871 | -0.0131 |
| 1974 | 0.0799 | 0.1234 | -0.0387 |
| 1975 | 0.0587 | 0.0694 | -0.0100 |
| 1976 | 0.0507 | 0.0486 | 0.0020 |
| 1977 | 0.0545 | 0.0670 | -0.0117 |
| 1978 | 0.0764 | 0.0902 | -0.0127 |
| 1979 | 0.1056 | 0.1329 | -0.0241 |
| 1980 | 0.1210 | 0.1252 | -0.0037 |
|  | 0.6197 | 0.7438 | -0.1120 |

a. The average return for T-bills over this period was:

Average return $=0.619$ / 8
Average return $=.0775$ or $7.75 \%$
And the average inflation rate was:
Average inflation $=0.7438 / 8$
Average inflation $=.0930$ or $9.30 \%$
b. Using the equation for variance, we find the variance for T-bills over this period was:
$\begin{aligned} \text { Variance }= & 1 / 7\left[(.0729-.0775)^{2}+(.0799-.0775)^{2}+(.0587-.0775)^{2}+(.0507-.0775)^{2}\right. \\ & \left.+(.0545-.0775)^{2}+(.0764-.0775)^{2}+(.1056-.0775)^{2}+(.1210-.0775)^{2}\right]\end{aligned}$
Variance $=0.000616$
And the standard deviation for T-bills was:
Standard deviation $=(0.000616)^{1 / 2}$
Standard deviation $=0.0248$ or $2.48 \%$

The variance of inflation over this period was:

```
Variance \(=1 / 7\left[(.0871-.0930)^{2}+(.1234-.0930)^{2}+(.0694-.0930)^{2}+(.0486-.0930)^{2}\right.\)
        \(\left.+(.0670-.0930)^{2}+(.0902-.0930)^{2}+(.1329-.0930)^{2}+(.1252-.0930)^{2}\right]\)
Variance \(=0.000971\)
```

And the standard deviation of inflation was:
Standard deviation $=(0.000971)^{1 / 2}$
Standard deviation $=0.0312$ or $3.12 \%$
c. The average observed real return over this period was:

Average observed real return $=-.1122 / 8$
Average observed real return $=-.0140$ or $-1.40 \%$
d. The statement that T-bills have no risk refers to the fact that there is only an extremely small chance of the government defaulting, so there is little default risk. Since T-bills are short term, there is also very limited interest rate risk. However, as this example shows, there is inflation risk, i.e. the purchasing power of the investment can actually decline over time even if the investor is earning a positive return.
23. To find the return on the coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has six years to maturity, so the price today is:
$\mathrm{P}_{1}=\$ 80\left(\mathrm{PVIFA}_{7 \sigma, 9}\right)+\$ 1,000 / 1.07^{9}$
$\mathrm{P}_{1}=\$ 1,065.15$
You received the coupon payments on the bond, so the nominal return was:
$\mathrm{R}=(\$ 1,065.15-1,028.50+80) / \$ 1,028.50$
$\mathrm{R}=.1134$ or $11.34 \%$
And using the Fisher equation to find the real return, we get:
$\mathrm{r}=(1.1134 / 1.048)-1$
$\mathrm{r}=.0624$ or $6.24 \%$
24. Looking at the long-term government bond return history in Figure 10.10, we see that the mean return was 5.8 percent, with a standard deviation of 9.2 percent. In the normal probability distribution, approximately $2 / 3$ of the observations are within one standard deviation of the mean. This means that $1 / 3$ of the observations are outside one standard deviation away from the mean. Or:
$\operatorname{Pr}(\mathrm{R}<-3.4$ or $\mathrm{R}>15.0) \approx 1 / 3$

## B-200 SOLUTIONS

But we are only interested in one tail here, that is, returns less than -3.0 percent, so:
$\operatorname{Pr}(\mathrm{R}<-3.5) \approx 1 / 6$
You can use the z-statistic and the cumulative normal distribution table to find the answer as well. Doing so, we find:
$\mathrm{z}=(\mathrm{X}-\mu) / \sigma$
$\mathrm{z}=(-3.5 \%-5.8) / 9.2 \%=-1.00$
Looking at the z-table, this gives a probability of $15.87 \%$, or:
$\operatorname{Pr}(\mathrm{R}<-3.5) \approx .1587$ or $15.87 \%$
The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:
$95 \%$ level: $R \in \mu \pm 2 \sigma=5.8 \% \pm 2(9.2 \%)=-12.60 \%$ to $24.20 \%$
The range of returns you would expect to see 99 percent of the time is the mean plus or minus 3 standard deviations, or:
$99 \%$ level: $R \in \mu \pm 3 \sigma=5.8 \% \pm 3(9.2 \%)=-21.80 \%$ to $33.40 \%$
25. The mean return for small company stocks was 17.4 percent, with a standard deviation of 32.7 percent. Doubling your money is a $100 \%$ return, so if the return distribution is normal, we can use the z -statistic. So:
$\mathrm{z}=(\mathrm{X}-\mu) / \sigma$
$\mathrm{z}=(100 \%-17.4) / 32.7 \%=2.526$ standard deviations above the mean
This corresponds to a probability of $\approx 0.577 \%$, or less than once every 100 years. Tripling your money would be:
$\mathrm{z}=(200 \%-17.4) / 32.7 \%=5.584$ standard deviations above the mean.
This corresponds to a probability of (much) less than $0.5 \%$, or once every 200 years. The actual answer is $\approx .00000117 \%$, or about once every 1 million years.
26. It is impossible to lose more than 100 percent of your investment. Therefore, return distributions are truncated on the lower tail at -100 percent, and cannot truly follow a normal distribution.

## Challenge

27. Using the z -statistic, we find:
$\mathrm{z}=(\mathrm{X}-\mu) / \sigma$
$\mathrm{z}=(0 \%-12.3) / 20.1 \%=-0.6119$
$\operatorname{Pr}(\mathrm{R} \leq 0) \approx 27.03 \%$
28. For each of the questions asked here, we need to use the $z$-statistic, which is:
$\mathrm{z}=(\mathrm{X}-\mu) / \sigma$
a. $\mathrm{z}_{1}=(10 \%-6.2) / 8.5 \%=0.4471$

This z-statistic gives us the probability that the return is less than 10 percent, but we are looking for the probability the return is greater than 10 percent. Given the symmetry of the normal distribution, and the fact that the total probability is 100 percent (or 1 ), the probability of a return greater than 10 percent is 1 minus the probability of a return less than 10 percent. Using the cumulative normal distribution table, we get:
$\operatorname{Pr}(\mathrm{R} \geq 10 \%)=1-\operatorname{Pr}(\mathrm{R} \leq 10 \%)=1-.6726 \approx 32.74 \%$
For a return greater than 0 percent:
$\mathrm{z}_{2}=(0 \%-6.2) / 8.5=-0.7294$
$\operatorname{Pr}(\mathrm{R} \geq 10 \%)=1-\operatorname{Pr}(\mathrm{R} \leq 10 \%)=1-.7673 \approx 23.29 \%$
b. The probability that T-bill returns will be greater than 10 percent is:
$\mathrm{z}_{3}=(10 \%-3.8) / 3.1 \%=2$
$\operatorname{Pr}(\mathrm{R} \geq 10 \%)=1-\operatorname{Pr}(\mathrm{R} \leq 10 \%)=1-.9772 \approx 2.28 \%$
And the probability that T-bill returns will be less than 0 percent is:
$\mathrm{z}_{4}=(0 \%-3.8) / 3.1 \%=-1.2258$
$\operatorname{Pr}(\mathrm{R} \leq 0) \approx 11.01 \%$

## B-202 SOLUTIONS

c. The probability that the return on long-term corporate bonds will be less than -4.18 percent is:
$z_{5}=(-4.18 \%-6.2) / 8.5 \%=-1.2212$
$\operatorname{Pr}(\mathrm{R} \leq-4.18 \%) \approx 11.10 \%$
And the probability that T-bill returns will be greater than 10.32 percent is:
$\mathrm{z}_{6}=(10.32 \%-3.8) / 3.1 \%=2.1032$
$\operatorname{Pr}(\mathrm{R} \geq 10.32 \%)=1-\operatorname{Pr}(\mathrm{R} \leq 10.32 \%)=1-.9823 \approx 1.77 \%$

## CHAPTER 11 <br> RISK AND RETURN

## Answers to Concepts Review and Critical Thinking Questions

1. Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unsystematic portion of the total risk can be eliminated at little cost. On the other hand, there are systematic risks that affect all investments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.
2. If the market expected the growth rate in the coming year to be 2 percent, then there would be no change in security prices if this expectation had been fully anticipated and priced. However, if the market had been expecting a growth rate different than 2 percent and the expectation was incorporated into security prices, then the government's announcement would most likely cause security prices in general to change; prices would typically drop if the anticipated growth rate had been more than 2 percent, and prices would typically rise if the anticipated growth rate had been less than 2 percent.
3. a. systematic
b. unsystematic
c. both; probably mostly systematic
d. unsystematic
e. unsystematic
f. systematic
4. a. This is a systematic risk; market prices in general will most likely decline.
$b$. This is a firm specific risk; the company price will most likely stay constant.
c. This is a systematic risk; market prices in general will most likely stay constant.
d. This is a firm specific risk; the company price will most likely decline.
e. This is a systematic risk; market prices in general will most likely stay constant.
5. No to both questions. The portfolio expected return is a weighted average of the asset returns, so it must be less than the largest asset return and greater than the smallest asset return.
6. False. The variance of the individual assets is a measure of the total risk. The variance and expected return on a well-diversified portfolio are functions of systematic risk only.
7. Yes, the standard deviation can be less than that of every asset in the portfolio. However, $\beta$ cannot be less than the smallest beta because $\beta_{\mathrm{P}}$ is a weighted average of the individual asset betas.

## B-204 SOLUTIONS

8. Yes. It is possible, in theory, to construct a zero beta portfolio of risky assets whose return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument.
9. Such layoffs generally occur in the context of corporate restructurings. To the extent that the market views a restructuring as value-creating, stock prices will rise. So, it's not the layoffs per se that are being cheered on but the cost savings associated with the layoffs. Nonetheless, Wall Street does encourage corporations to takes actions to create value, even if such actions involve layoffs.
10. Earnings contain information about recent sales and costs. This information is useful for projecting future growth rates and cash flows. Thus, unexpectedly low earnings often lead market participants to reduce estimates of future growth rates and cash flows; lower prices are the result. The reverse is often true for unexpectedly high earnings.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

Total value $=90(\$ 84)+50(\$ 58)$
Total value $=\$ 10,460$
The portfolio weight for each stock is:
Weight $_{\mathrm{A}}=90(\$ 84) / \$ 10,460$
Weight $_{\mathrm{A}}=.7228$
Weight $_{\mathrm{B}}=50(\$ 58) / \$ 10,460$
Weight $_{\mathrm{B}}=.2772$
2. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

$$
\begin{aligned}
& \text { Total value }=\$ 700+2,500 \\
& \text { Total value }=\$ 3,200
\end{aligned}
$$

So, the expected return of this portfolio is:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(\$ 700 / \$ 3,200)(.10)+(\$ 2,500 / \$ 3,200)(.16) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1469 \text { or } 14.69 \%
\end{aligned}
$$

3. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.20(.10)+.45(.14)+.35(.16)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1390$ or $13.90 \%$
4. Here, we are given the expected return of the portfolio and the expected return of each asset in the portfolio, and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal $1(100 \%)$, the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1425=.16 \mathrm{w}_{\mathrm{x}}+.11\left(1-\mathrm{w}_{\mathrm{x}}\right)$
We can now solve this equation for the weight of Stock X as:

$$
\begin{aligned}
& .1425=.16 \mathrm{w}_{\mathrm{X}}+.11-.11 \mathrm{w}_{\mathrm{X}} \\
& .0325=.05 \mathrm{w}_{\mathrm{X}} \\
& \mathrm{w}_{\mathrm{X}}=0.6500
\end{aligned}
$$

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:
Investment in $X=0.6500(\$ 10,000)=\$ 6,500$
And the dollar amount invested in Stock Y is:
Investment in $Y=(1-0.6500)(\$ 10,000)=\$ 3,500$
5. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the asset is:
$\mathrm{E}(\mathrm{R})=.15(-.09)+.85(.18)$
$R(R)=.1395$ or $13.95 \%$
6. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the asset is:
$\mathrm{E}(\mathrm{R})=.20(-.07)+.55(.13)+.25(.30)$
$\mathrm{E}(\mathrm{R})=.1325$ or $13.25 \%$
7. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of each stock asset is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=.15(.01)+.55(.09)+.30(.14)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=.0930$ or $9.30 \%$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)=.15(-.25)+.55(.15)+.30(.38)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)=.1590$ or $15.90 \%$

## B-206 SOLUTIONS

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of each stock is:
$\sigma_{\mathrm{A}}{ }^{2}=.15(.01-.0930)^{2}+.55(.09-.0930)^{2}+.30(.14-.0930)^{2}$
$\sigma_{\mathrm{A}}{ }^{2}=.00170$
$\sigma_{\mathrm{A}}=(.00170)^{1 / 2}$
$\sigma_{\mathrm{A}}=.0412$ or $4.12 \%$
$\sigma_{\mathrm{B}}{ }^{2}=.15(-.25-.1590)^{2}+.55(.15-.1590)^{2}+.30(.38-.1590)^{2}$
$\sigma_{B}{ }^{2}=.03979$
$\sigma_{B}=(.03979)^{1 / 2}$
$\sigma_{B}=.1995$ or $19.95 \%$
8. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the asset is:
$\mathrm{E}(\mathrm{R})=.20(.09)+.35(.13)+.45(.19)$
$\mathrm{E}(\mathrm{R})=.1490$ or $14.90 \%$
9. $a$. To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

Boom: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(.08+.02+.33) / 3$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1433$ or $14.33 \%$
Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(.14+.24-.06) / 3$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1067$ or $10.67 \%$
This is equivalent to multiplying the weight of each asset ( $1 / 3$ or .3333 ) times its expected return and summing the results, which gives:

Boom: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=1 / 3(.08)+1 / 3(.02)+1 / 3(.33)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1433$ or $14.33 \%$
Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=1 / 3(.14)+1 / 3(.24)+1 / 3(-.06)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1067$ or $10.67 \%$

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.65(.1433)+.35(.1067) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1305 \text { or } 13.05 \%
\end{aligned}
$$

b. This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$
\text { Boom: } \begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.20(.08)+.20(.02)+.60(.33) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.2180 \text { or } 21.80 \%
\end{aligned}
$$

Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.20(.14)+.20(.24)+.60(-.06)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0400 \text { or } 4.00 \%
$$

And the expected return of the portfolio is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.65(.2180)+.35(.0400)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1557$ or $15.57 \%$
To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance of the portfolio is:

$$
\begin{aligned}
& \sigma_{\mathrm{p}}^{2}=.65(.2180-.1557)^{2}+.35(.0400-.1557)^{2} \\
& \sigma_{\mathrm{p}}^{2}=.00721
\end{aligned}
$$

10. a. This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.30(.30)+.40(.45)+.30(.33)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.3690 \text { or } 36.90 \%
$$

Good: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.30(.12)+.40(.10)+.30(.15)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1210 \text { or } 12.10 \%
$$

Poor: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.30(.01)+.40(-.15)+.30(-.05)$ $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=-.0720$ or $-7.20 \%$

Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.30(-.20)+.40(-.30)+.30(-.09)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=-.2070 \text { or }-20.70 \%
$$

## B-208 SOLUTIONS

And the expected return of the portfolio is:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.15(.3690)+.45(.1210)+.35(-.0720)+.05(-.2070) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0743 \text { or } 7.43 \%
\end{aligned}
$$

b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio is:

$$
\left.\begin{array}{rl}
\sigma_{\mathrm{p}}^{2}= & .15(.3690-.0743)^{2}+.45(.1210-.0743)^{2}+.35(-.0720-.0743)^{2} \\
& \quad .05(-.2070-.0743)^{2}
\end{array} \sigma_{\mathrm{p}}^{2}=.02546\right)
$$

11. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:
$\beta_{\mathrm{p}}=.25(.73)+.20(.86)+.45(1.25)+.10(1.84)$
$\beta_{\mathrm{p}}=1.10$
12. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market, it must have the same beta as the market. Since the beta of the market is one, we know the beta of our portfolio is one. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:
$\beta_{p}=1.0=1 / 3(0)+1 / 3(1.65)+1 / 3\left(\beta_{\mathrm{X}}\right)$
Solving for the beta of Stock X, we get:
$\beta_{\mathrm{X}}=1.35$
13. The CAPM states the relationship between the risk of an asset and its expected return. The CAPM is:
$E\left(R_{i}\right)=R_{f}+\left[E\left(R_{M}\right)-R_{f}\right] \times \beta_{i}$
Substituting the values we are given, we find:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=.06+(.13-.06)(0.90)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=.1230$ or $12.30 \%$
14. We are given the values for the CAPM except for the $\beta$ of the stock. We need to substitute these values into the CAPM, and solve for the $\beta$ of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=.17=.055+.08 \beta_{\mathrm{i}}$
$\beta_{i}=1.438$
15. Here, we need to find the expected return of the market, using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=.17=.055+\left[\mathrm{E}\left(\mathrm{R}_{\mathrm{M}}\right)-.055\right](1.45)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{M}}\right)=.1343$ or $13.43 \%$
16. Here, we need to find the risk-free rate, using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:
```
\(\mathrm{E}\left(\mathrm{R}_{\mathrm{i}}\right)=.1190=\mathrm{R}_{\mathrm{f}}+\left(.13-\mathrm{R}_{\mathrm{f}}\right)(.85)\)
\(.1190=\mathrm{R}_{\mathrm{f}}+.1105-.85 \mathrm{R}_{\mathrm{f}}\)
\(\mathrm{R}_{\mathrm{f}}=.0567\) or \(5.67 \%\)
```

17. a. Again, we have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(.16+.065) / 2 \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1113 \text { or } 11.13 \%
\end{aligned}
$$

b. We need to find the portfolio weights that result in a portfolio with a $\beta$ of 0.75 . We know the $\beta$ of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:
$\beta_{\mathrm{p}}=0.80=\mathrm{w}_{\mathrm{s}}(1.4)+\left(1-\mathrm{w}_{\mathrm{s}}\right)(0)$
$0.80=1.4 \mathrm{w}_{\mathrm{S}}+0-0 \mathrm{w}_{\mathrm{s}}$
$\mathrm{w}_{\mathrm{S}}=0.80 / 1.4$
$\mathrm{w}_{\mathrm{S}}=.5714$
And, the weight of the risk-free asset is:

$$
\begin{aligned}
& \mathrm{w}_{\mathrm{Rf}}=1-.5714 \\
& \mathrm{w}_{\mathrm{Rf}}=.4286
\end{aligned}
$$

## B-210 SOLUTIONS

c. We need to find the portfolio weights that result in a portfolio with an expected return of 12 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

```
\(\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.12=.16 \mathrm{w}_{\mathrm{S}}+.065\left(1-\mathrm{w}_{\mathrm{S}}\right)\)
\(.12=.16 w_{\mathrm{S}}+.065-.065 \mathrm{w}_{\mathrm{S}}\)
\(\mathrm{w}_{\mathrm{S}}=.5897\)
```

So, the $\beta$ of the portfolio will be:
$\beta_{\mathrm{p}}=.5897(1.4)+(1-.5897)(0)$
$\beta_{\mathrm{p}}=0.826$
d. Solving for the $\beta$ of the portfolio as we did in part $a$, we find:
$\beta_{\mathrm{p}}=2.8=\mathrm{w}_{\mathrm{S}}(1.4)+\left(1-\mathrm{w}_{\mathrm{S}}\right)(0)$
$\mathrm{w}_{\mathrm{S}}=2.8 / 1.4$
$\mathrm{w}_{\mathrm{S}}=2$
$\mathrm{w}_{\mathrm{Rf}}=1-2$
$\mathrm{w}_{\mathrm{Rf}}=-1$
The portfolio is invested $200 \%$ in the stock and $-100 \%$ in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.
18. First, we need to find the $\beta$ of the portfolio. The $\beta$ of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, the $\beta$ of the portfolio is:
$\beta_{\mathrm{p}}=\mathrm{w}_{\mathrm{W}}(1.25)+\left(1-\mathrm{w}_{\mathrm{W}}\right)(0)=1.25 \mathrm{w}_{\mathrm{W}}$
So, to find the $\beta$ of the portfolio for any weight of the stock, we simply multiply the weight of the stock times its $\beta$.

Even though we are solving for the $\beta$ and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock, and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{W}}\right)=.165=.06+\mathrm{MRP}(1.25)$
$\mathrm{MRP}=.105 / 1.25=.0840$ or $8.40 \%$

So, now we know the CAPM equation for any stock is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.06+.0840 \beta_{\mathrm{p}}$

The slope of the SML is equal to the market risk premium, which is 0.0840 . Using these equations to fill in the table, we get the following results:

| $\mathrm{w}_{\mathrm{W}}$ | $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)$ | $\beta_{\mathrm{p}}$ |
| :---: | :---: | :---: |
| $0 \%$ | .0600 | 0 |
| 25 | .0863 | 0.313 |
| 50 | .1125 | 0.625 |
| 75 | .1388 | 0.938 |
| 100 | .1650 | 1.250 |
| 125 | .1913 | 1.563 |
| 150 | .2175 | 1.875 |

19. There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{Y}}\right)=.055+.080(1.50)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{Y}}\right)=.1750$ or $17.50 \%$
It is given in the problem that the expected return of Stock Y is 16 percent, but according to the CAPM, the return of the stock based on its level of risk, the expected return should be 17.50 percent. This means the stock return is too low, given its level of risk. Stock Y plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 17.50 percent. For Stock Z, we find:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{Z}}\right)=.055+.080(0.70) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{Z}}\right)=.1110 \text { or } 11.10 \%
\end{aligned}
$$

The return given for Stock Z is 11.5 percent, but according to the CAPM the expected return of the stock should be 11.10 percent based on its level of risk. Stock Z plots above the SML and is undervalued. In other words, its price must increase to decrease the expected return to 11.10 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio. The reward-to-risk ratio is the risk premium of the asset divided by its $\beta$. We are given the market risk premium, and we know the $\beta$ of the market is one, so the reward-to-risk ratio for the market is 0.08 , or 8 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

Reward-to-risk ratio $\mathrm{Y}=(.16-.055) / 1.50$
Reward-to-risk ratio $\mathrm{Y}=.0700$

## B-212 SOLUTIONS

The reward-to-risk ratio for Stock Y is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

Reward-to-risk ratio Z = (. 115 - .055) / . 70
Reward-to-risk ratio $\mathrm{Z}=.0857$
The reward-to-risk ratio for Stock Z is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.
20. We need to set the reward-to-risk ratios of the two assets equal to each other, which is:
$\left(.16-\mathrm{R}_{\mathrm{f}}\right) / 1.50=\left(.115-\mathrm{R}_{\mathrm{f}}\right) / 0.70$
We can cross multiply to get:
$0.70\left(.16-\mathrm{R}_{\mathrm{f}}\right)=1.50\left(.115-\mathrm{R}_{\mathrm{f}}\right)$
Solving for the risk-free rate, we find:
$0.112-0.70 \mathrm{R}_{\mathrm{f}}=0.1725-1.50 \mathrm{R}_{\mathrm{f}}$
$\mathrm{R}_{\mathrm{f}}=.0756$ or $7.56 \%$
21. For a portfolio that is equally invested in large-company stocks and long-term corporate bonds:
$\mathrm{R}=(12.30 \%+6.20 \%) / 2$
$\mathrm{R}=9.25 \%$
For a portfolio that is equally invested in small stocks and Treasury bills:
$\mathrm{R}=(17.4 \%+3.8 \%) / 2$
$\mathrm{R}=10.60 \%$
22. Here, we are given the expected return of the portfolio and the expected return of the assets in the portfolio and are asked to calculate the dollar amount of each asset in the portfolio. So, we need to find the weight of each asset in the portfolio. Since we know the total weight of the assets in the portfolio must equal 1 (or $100 \%$ ), we can find the weight of each asset as:
$\mathrm{E}\left[\mathrm{R}_{\mathrm{p}}\right]=.12=.16 \mathrm{w}_{\mathrm{H}}+.095\left(1-\mathrm{w}_{\mathrm{H}}\right)$
$\mathrm{w}_{\mathrm{H}}=0.3846$
$\mathrm{w}_{\mathrm{L}}=1-\mathrm{w}_{\mathrm{H}}$
$\mathrm{w}_{\mathrm{L}}=1-.3846$
$\mathrm{w}_{\mathrm{L}}=.6154$

So, the dollar investment in each asset is the weight of the asset times the value of the portfolio, so the dollar investment in each asset must be:

Investment in $\mathrm{H}=0.3846(\$ 250,000)$
Investment in $\mathrm{H}=\$ 96,153.85$

Investment in $\mathrm{L}=0.6154(\$ 250,000)$
Investment in $\mathrm{L}=\$ 153,846.15$
23. $a$. To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

Boom: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(.04+.03+.26) / 3$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1100$ or $11.00 \%$
Bust: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=(.12+.18-.02) / 3$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0933$ or $9.33 \%$
This is equivalent to multiplying the weight each asset ( $1 / 3$ or .3333 ) times its expected return and summing the results, which gives:

Boom: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=1 / 3(.04)+1 / 3(.03)+1 / 3(.26)$ $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1100$ or $11.00 \%$

Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=1 / 3(.12)+1 / 3(.18)+1 / 3(-.02)$ $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0933$ or $9.33 \%$

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.75(.1100)+.25(.0933) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1058 \text { or } 10.58 \%
\end{aligned}
$$

b. This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$
\begin{array}{ll}
\text { Boom: } & \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.25(.04)+.25(.03)+.50(.26) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1475 \text { or } 14.75 \% \\
\text { Bust: } & \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.25(.12)+.25(.18)+.50(-.02) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0650 \text { or } 6.50 \%
\end{array}
$$

## B-214 SOLUTIONS

And the expected return of the portfolio is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.75(.1475)+.25(.0650)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1269$ or $12.69 \%$
To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance of the portfolio is:

$$
\begin{aligned}
& \sigma_{\mathrm{p}}^{2}=.75(.1475-.1269)^{2}+.25(.0650-.1269)^{2} \\
& \sigma_{\mathrm{p}}{ }^{2}=.00128
\end{aligned}
$$

And the standard deviation of the portfolio is:

$$
\begin{aligned}
& \sigma_{\mathrm{P}}=(.00128)^{1 / 2} \\
& \sigma^{\mathrm{P}}=.0357 \text { or } 3.57 \%
\end{aligned}
$$

24. a. This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: $\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.60(.18)+.20(.35)+.20(.20)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.2180$ or $21.80 \%$
Good: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.60(.11)+.20(.15)+.20(.11)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1180$ or $11.80 \%$
Poor: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.60(.06)+.20(-.05)+.20(.02)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0300 \text { or } 3.00 \%
$$

Bust: $\quad \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.60(.01)+.20(-.40)+.20(-.08)$

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=-.0900 \text { or }-9.00 \%
$$

And the expected return of the portfolio is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.10(.2180)+.60(.1180)+.20(.0300)+.10(-.0900)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.0896$ or $8.96 \%$
b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio are:

$$
\left.\begin{array}{rl}
\sigma_{\mathrm{p}}^{2}= & .10(.2180-.0896)^{2}+.60(.1180-.0896)^{2}+.20(.0300-.0896)^{2} \\
& +.10(-.0900-.0896)^{2} \\
\sigma_{\mathrm{p}}^{2}=.006069
\end{array}\right] .
$$

25. To find the expected return of the portfolio we first need to find the weight of each asset in the portfolio. The weights of the assets sum to 1 (or $100 \%$ ), so we can solve for the weights, using the betas of the each asset and the beta of the portfolio. Doing so, we find:
$\beta_{\mathrm{p}}=1=\mathrm{w}_{\mathrm{J}}(1.35)+\left(1-\mathrm{w}_{\mathrm{J}}\right)(0.80)$
$\mathrm{w}_{\mathrm{J}}=.3636$
$\mathrm{w}_{\mathrm{K}}=1-.3636$
$\mathrm{w}_{\mathrm{K}}=.6364$
So, the expected return of the portfolio is:
$\mathrm{E}\left[\mathrm{R}_{\mathrm{P}}\right]=.3636(.17)+.6364(.10)$
$\mathrm{E}\left[\mathrm{R}_{\mathrm{P}}\right]=.1255$ or $12.55 \%$
26. To find the expected return of the portfolio, we first need to find the weight of each asset in the portfolio. The weight of each asset is the dollar investment of that asset divided by the total dollar value of the portfolio, so:

Portfolio value $=500(\$ 41)+900(\$ 27)+250(\$ 65)+625(\$ 49)$
Portfolio value $=\$ 91,675$
And the weight of each asset in the portfolio is:

$$
\begin{aligned}
& \mathrm{w}_{\mathrm{W}}=500(\$ 41) / \$ 91,675 \\
& \mathrm{w}_{\mathrm{W}}=.2236 \\
& \mathrm{w}_{\mathrm{X}}=900(\$ 27) / \$ 91,675 \\
& \mathrm{w}_{\mathrm{X}}=.2651 \\
& \mathrm{w}_{\mathrm{Y}}=250(\$ 65) / \$ 91,675 \\
& \mathrm{w}_{\mathrm{Y}}=.1773 \\
& \mathrm{w}_{\mathrm{Z}}=625(\$ 49) / \$ 91,675 \\
& \mathrm{w}_{\mathrm{Z}}=.3341
\end{aligned}
$$

## B-216 SOLUTIONS

With the weight of each asset, we can find the expected return of the portfolio, which is:

$$
\begin{aligned}
& \mathrm{E}\left[\mathrm{R}_{\mathrm{P}}\right]=.2236(.12)+.2651(.16)+.1773(.14)+.3341(.15) \\
& \mathrm{E}\left[\mathrm{R}_{\mathrm{P}}\right]=.1442 \text { or } 14.42 \%
\end{aligned}
$$

## Intermediate

27. We know the expected return of the portfolio and of each asset, but only one portfolio weight. We need to recognize that the weight of the risk-free asset is one minus the weight of the other two assets. Mathematically, the expected return of the portfolio is:
```
\(\mathrm{E}\left[\mathrm{R}_{\mathrm{p}}\right]=.12=.50(.16)+\mathrm{w}_{\mathrm{F}}(.105)+\left(1-.50-\mathrm{w}_{\mathrm{F}}\right)(.06)\)
\(.12=.50(.16)+\mathrm{w}_{\mathrm{F}}(.10)+.06-.03-.06 \mathrm{w}_{\mathrm{F}}\)
\(\mathrm{w}_{\mathrm{F}}=.2222\)
```

So, the weight of the risk-free asset is:
$\mathrm{w}_{\mathrm{Rf}}=1-.50-.2222$
$\mathrm{w}_{\mathrm{Rf}}=.2778$
And the amount of Stock F to buy is:
Amount of stock F to buy $=.2222(\$ 100,000)$
Amount of stock F to buy $=\$ 22,222.22$
28. a. We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$
\begin{array}{ll}
\text { Boom: } & \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.40(.05)+.40(.25)+.20(.60) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.2400 \text { or } 24.00 \% \\
\text { Normal: } & \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.40(.09)+.40(.12)+.20(.20) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1240 \text { or } 12.40 \% \\
\text { Bust: } & \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.40(.12)+.40(-.13)+.20(-.40) \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=-.0840 \text { or }-8.40 \%
\end{array}
$$

And the expected return of the portfolio is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.25(.2400)+.55(.1240)+.20(-.0840)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.1114$ or $11.14 \%$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio are:

$$
\begin{aligned}
& \sigma_{p}^{2}=.25(.2400-.1114)^{2}+.55(.1240-.1114)^{2}+.20(-.0840-.1114)^{2} \\
& \sigma_{p}^{2}=.01186 \\
& \sigma_{p}=(.01186)^{1 / 2} \\
& \sigma_{p}=.1089 \text { or } 10.89 \%
\end{aligned}
$$

b. The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:

```
RP
RP
RP
```

29. We know that the reward-to-risk ratios for all assets must be equal. This can be expressed as:
$\left[\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)-\mathrm{R}_{\mathrm{f}}\right] / \beta_{\mathrm{A}}=\left[\mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)-\mathrm{R}_{\mathrm{f}}\right] / \beta_{\mathrm{B}}$
The numerator of each equation is the risk premium of the asset, so:
$\mathrm{RP}_{\mathrm{A}} / \beta_{\mathrm{A}}=\mathrm{RP}_{\mathrm{B}} / \beta_{\mathrm{B}}$
We can rearrange this equation to get:
$\beta_{\mathrm{B}} / \beta_{\mathrm{A}}=\mathrm{RP}_{\mathrm{B}} / \mathrm{RP}_{\mathrm{A}}$
If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.
30. Since the portfolio is as risky as the market, the $\beta$ of the portfolio must be equal to one. We also know the $\beta$ of the risk-free asset is zero. We can use the equation for the $\beta$ of a portfolio to find the weight of the third stock. Doing so, we find:
$\beta_{\mathrm{p}}=1.0=\mathrm{w}_{\mathrm{A}}(.85)+\mathrm{w}_{\mathrm{B}}(1.20)+\mathrm{w}_{\mathrm{C}}(1.45)+\mathrm{w}_{\mathrm{Rf}}(0)$
Solving for the weight of Stock C, we find:
$\mathrm{w}_{\mathrm{C}}=.288966$
So, the dollar investment in Stock C must be:
Invest in Stock C = .288966(\$500,000)
Invest in Stock C = \$144,482.76

We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

$$
\begin{aligned}
& \mathrm{w}_{\mathrm{A}}=\$ 130,000 / \$ 500,000 \\
& \mathrm{w}_{\mathrm{A}}=.26 \\
& \mathrm{w}_{\mathrm{B}}=\$ 150,000 / \$ 500,000 \\
& \mathrm{w}_{\mathrm{B}}=.30
\end{aligned}
$$

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

```
\(1=\mathrm{w}_{\mathrm{A}}+\mathrm{w}_{\mathrm{B}}+\mathrm{w}_{\mathrm{C}}+\mathrm{w}_{\mathrm{Rf}}=1-.26-.30-.288966-\mathrm{w}_{\mathrm{Rf}}\)
\(\mathrm{w}_{\mathrm{Rf}}=.151034\)
```

So, the dollar investment in the risk-free asset must be:
Invest in risk-free asset $=.151034(\$ 500,000)$
Invest in risk-free asset $=\$ 75,517.24$
31. We are given the expected return of the assets in the portfolio. We also know the sum of the weights of each asset must be equal to one. So, the weight of the risk-free asset is one minus the weight of Stock Y and the weight of Stock X . Using this relationship, we can express the expected return of the portfolio as:

```
\(\mathrm{E}\left(\mathrm{R}_{\mathrm{p}}\right)=.185=\mathrm{w}_{\mathrm{X}}(.172)+\mathrm{w}_{\mathrm{Y}}(.136)\)
\(.185=w_{\mathrm{X}}(.172)+\left(1-\mathrm{w}_{\mathrm{X}}\right)(.136)\)
\(.185=.172 \mathrm{w}_{\mathrm{X}}+.136-.136 \mathrm{w}_{\mathrm{X}}\)
\(.049=.036 \mathrm{w}_{\mathrm{X}}\)
\(\mathrm{w}_{\mathrm{X}}=1.36111\)
```

And the weight of Stock Y is:
$\mathrm{w}_{\mathrm{Y}}=1-1.36111$
$\mathrm{w}_{\mathrm{Y}}=-.36111$
The amount to invest in Stock Y is:
Investment in Stock Y $=-.36111$ ( $\$ 100,000)$
Investment in Stock Y $=-\$ 36,111.11$
A negative portfolio weight means that you short sell the stock. If you are not familiar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value.

To find the beta of the portfolio, we can multiply the portfolio weight of each asset times its beta and sum. So, the beta of the portfolio is:
$\beta_{\mathrm{P}}=1.36111(1.40)+(-.36111)(0.95)$
$\beta_{\mathrm{P}}=1.56$
32. The amount of systematic risk is measured by the $\beta$ of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset, we can use the CAPM to solve for the $\beta$ of the asset. The expected return of Stock I is:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{I}}\right)=.30(.02)+.40(.32)+.30(.18)$
$\mathrm{E}\left(\mathrm{R}_{\mathrm{I}}\right)=.1880$ or $18.80 \%$
Using the CAPM to find the $\beta$ of Stock I, we find:
$.1880=.04+.12 \beta_{I}$
$\beta_{\mathrm{I}}=1.23$
The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock's variance, we find:
$\sigma_{\mathrm{I}}^{2}=.30(.02-.1880)^{2}+.40(.32-.1880)^{2}+.30(.18-.1880)^{2}$
$\sigma_{\mathrm{I}}{ }^{2}=.01546$
$\sigma_{\mathrm{I}}=(.01546)^{1 / 2}$
$\sigma_{\mathrm{I}}=.1243$ or $11.43 \%$
Using the same procedure for Stock II, we find the expected return to be:
$\mathrm{E}\left(\mathrm{R}_{\mathrm{II}}\right)=.30(-.20)+.40(.12)+.30(.40)$
$\mathrm{E}\left(\mathrm{R}_{\text {II }}\right)=.1080$ or $10.80 \%$
Using the CAPM to find the $\beta$ of Stock II, we find:
$.1080=.04+.12 \beta_{\text {II }}$
$\beta_{\text {II }}=0.57$
And the standard deviation of Stock II is:
$\sigma_{\mathrm{II}}{ }^{2}=.30(-.20-.1080)^{2}+.40(.12-.1080)^{2}+.30(.40-.1080)^{2}$
$\sigma_{\text {II }}{ }^{2}=.05410$
$\sigma_{\text {II }}=(.05410)^{1 / 2}$
$\sigma_{\mathrm{II}}=.2326$ or $23.26 \%$
Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I's. Thus, I has more systematic risk, and II has more unsystematic and more total risk.

## B-220 SOLUTIONS

Since unsystematic risk can be diversified away, I is actually the "riskier" stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

## CHAPTER 12 COST OF CAPITAL

## Answers to Concepts Review and Critical Thinking Questions

1. It is the minimum rate of return the firm must earn overall on its existing assets. If it earns more than this, value is created.
2. Book values for debt are likely to be much closer to their market values than are book values for equity.
3. No. The cost of capital depends on the risk of the project, not the source of the money.
4. Interest expense is tax-deductible. There is no difference between pretax and aftertax equity costs.
5. The primary advantage of the DCF model is its simplicity. The method is disadvantaged in that (1) the model is applicable only to firms that actually pay dividends; many do not; (2) even if a firm does pay dividends, the DCF model requires a constant dividend growth rate forever; (3) the estimated cost of equity from this method is very sensitive to changes in g , which is a very uncertain parameter; and (4) the model does not explicitly consider risk, although risk is implicitly considered to the extent that the market has impounded the relevant risk of the stock into its market price. While the share price and most recent dividend can be observed in the market, the dividend growth rate must be estimated. Two common methods of estimating g are to use analysts' earnings and payout forecasts, or determine some appropriate average historical g from the firm's available data.
6. Two primary advantages of the SML approach are that the model explicitly incorporates the relevant risk of the stock, and the method is more widely applicable than is the DCF model, since the SML doesn't make any assumptions about the firm's dividends. The primary disadvantages of the SML method are (1) estimating three parameters: the risk-free rate, the expected return on the market, and beta, and (2) the method essentially uses historical information to estimate these parameters. The riskfree rate is usually estimated to be the yield on very short maturity T-bills and is hence observable; the market risk premium is usually estimated from historical risk premiums and is not directly observable. The stock beta, which is unobservable, is usually estimated either by determining some average historical beta from the firm and the market's return data, or using beta estimates provided by analysts and investment firms.
7. The appropriate aftertax cost of debt to the company is the interest rate it would have to pay if it were to issue new debt today. Hence, if the YTM on outstanding bonds of the company is observed, the company has an accurate estimate of its cost of debt. If the debt is privately placed, the firm could still estimate its cost of debt by (1) looking at the cost of debt for similar firms in similar risk classes, (2) looking at the average debt cost for firms with the same credit rating (assuming the firm's private debt is rated), or (3) consulting analysts and investment bankers. Even if the debt is publicly traded, an additional complication is when the firm has more than one issue outstanding; these issues rarely have the same yield because no two issues are ever completely homogeneous.

## B-222 SOLUTIONS

8. a. This only considers the dividend yield component of the required return on equity.
$b$. This is the current yield only, not the promised yield to maturity. In addition, it is based on the book value of the liability, and it ignores taxes.
c. Equity is inherently riskier than debt (except, perhaps, in the unusual case where a firm's assets have a negative beta). For this reason, the cost of equity exceeds the cost of debt. If taxes are considered in this case, it can be seen that at reasonable tax rates, the cost of equity does exceed the cost of debt.
9. $\quad R_{\text {Superior }}=.12+.75(.08)=.18$ or $18 \%$

Both should proceed. The appropriate discount rate does not depend on which company is investing; it depends on the risk of the project. Since Superior is in the business, it is closer to a pure play. Therefore, its cost of capital should be used. With an $18 \%$ cost of capital, the project has an NPV of \$1 million regardless of who takes it.
10. If the different operating divisions were in much different risk classes, then separate cost of capital figures should be used for the different divisions; the use of a single, overall cost of capital would be inappropriate. If the single hurdle rate were used, riskier divisions would tend to receive funds for investment projects, since their return would exceed the hurdle rate despite the fact that they may actually plot below the SML and hence be unprofitable projects on a risk-adjusted basis. The typical problem encountered in estimating the cost of capital for a division is that it rarely has its own securities traded on the market, so it is difficult to observe the market's valuation of the risk of the division. Two typical ways around this are to use a pure play proxy for the division, or to use subjective adjustments of the overall firm hurdle rate based on the perceived risk of the division.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. With the information given, we can find the cost of equity, using the dividend growth model. Using this model, the cost of equity is:
$\mathrm{R}_{\mathrm{E}}=[\$ 2.40(1.06) / \$ 48]+.06$
$\mathrm{R}_{\mathrm{E}}=.1130$ or $11.30 \%$
2. Here we have information to calculate the cost of equity, using the CAPM. The cost of equity is:

$$
\mathrm{R}_{\mathrm{E}}=.05+1.30(.13-.05)
$$

$$
\mathrm{R}_{\mathrm{E}}=.1540 \text { or } 15.40 \%
$$

3. We have the information available to calculate the cost of equity, using the CAPM and the dividend growth model. Using the CAPM, we find:
$\mathrm{R}_{\mathrm{E}}=.045+0.90(.08)=.1170$ or $11.70 \%$
And using the dividend growth model, the cost of equity is
$R_{E}=[\$ 2.60(1.05) / \$ 48]+.05=.1069$ or $10.69 \%$
Both estimates of the cost of equity seem reasonable. If we remember the historical return on large capitalization stocks, the estimate from the CAPM model is slightly lower than average, and the estimate from the dividend growth model is about one percent lower than the historical average, so we cannot definitively say one of the estimates is incorrect. Given this, we will use the average of the two, so:
$\mathrm{R}_{\mathrm{E}}=(.1170+.1069) / 2=.1119$ or $11.19 \%$
4. To use the dividend growth model, we first need to find the growth rate in dividends. So, the increase in dividends each year was:
$\mathrm{g}_{1}=(\$ 1.62-1.47) / \$ 1.47$
$\mathrm{g}_{1}=.1020$ or $10.20 \%$
$\mathrm{g}_{2}=(\$ 1.67-1.62) / \$ 1.62$
$\mathrm{g}_{2}=.0309$ or $3.09 \%$
$\mathrm{g}_{3}=(\$ 1.78-1.67) / \$ 1.67$
$\mathrm{g}_{3}=.0659$ or $6.59 \%$
$\mathrm{g}_{4}=(\$ 1.89-1.78) / \$ 1.78$
$\mathrm{g}_{4}=.0618$ or $6.18 \%$
So, the average arithmetic growth rate in dividends was:
$\mathrm{g}=(.1020+.0309+.0659+.0618) / 4$
$\mathrm{g}=.0651$ or $6.51 \%$
Using this growth rate in the dividend growth model, we find the cost of equity is:
$\mathrm{R}_{\mathrm{E}}=[\$ 1.89(1.0651) / \$ 65.00]+.0651$
$\mathrm{R}_{\mathrm{E}}=.0961$ or $9.61 \%$
Calculating the geometric growth rate in dividends, we find:
$\$ 1.89=\$ 1.47(1+\mathrm{g})^{4}$
$\mathrm{g}=.0648$ or $6.48 \%$

## B-224 SOLUTIONS

The cost of equity using the geometric dividend growth rate is:
$\mathrm{R}_{\mathrm{E}}=[\$ 1.89(1.0648) / \$ 65.00]+.0648$
$\mathrm{R}_{\mathrm{E}}=.0958$ or $9.58 \%$
5. The cost of preferred stock is the dividend payment divided by the price, so:
$\mathrm{R}_{\mathrm{P}}=\$ 6 / \$ 94$
$\mathrm{R}_{\mathrm{P}}=.0638$ or $6.38 \%$
6. The pretax cost of debt is the YTM of the company's bonds, so:

```
\(\mathrm{P}_{0}=\$ 930=\$ 28\left(\mathrm{PVIFA}_{\mathrm{R} \%, 14}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 14}\right)\)
\(\mathrm{R}=3.438 \%\)
\(\mathrm{YTM}=2 \times 3.438 \%\)
\(\mathrm{YTM}=6.88 \%\)
```

And the aftertax cost of debt is:
$\mathrm{R}_{\mathrm{D}}=.0688(1-.38)$
$\mathrm{R}_{\mathrm{D}}=.0426$ or $4.26 \%$
7. $a$. The pretax cost of debt is the YTM of the company's bonds, so:

```
\(\mathrm{P}_{0}=\$ 1,080=\$ 40\left(\mathrm{PVIFA}_{\mathrm{R} \%, 46}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 46}\right)\)
\(\mathrm{R}=3.639 \%\)
\(\mathrm{YTM}=2 \times 3.639 \%\)
\(\mathrm{YTM}=7.28 \%\)
```

b. The aftertax cost of debt is:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{D}}=.0728(1-.35) \\
& \mathrm{R}_{\mathrm{D}}=.0473 \text { or } 4.73 \%
\end{aligned}
$$

c. The after-tax rate is more relevant because that is the actual cost to the company.
8. The book value of debt is the total par value of all outstanding debt, so:

$$
B V_{D}=\$ 60,000,000+70,000,000
$$

$$
B V_{D}=\$ 130,000,000
$$

To find the market value of debt, we find the price of the bonds and multiply by the number of bonds. Alternatively, we can multiply the price quote of the bond times the par value of the bonds. Doing so, we find:
$M V_{D}=1.08(\$ 60,000,000)+.265(\$ 70,000,000)$
$\mathrm{MV}_{\mathrm{D}}=\$ 83,350,000$
The YTM of the zero coupon bonds is (Remember, even on zero coupon bonds, for consistency, the "payments" are assumed to be semiannual):
$\mathrm{P}_{\mathrm{Z}}=\$ 265=\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 40}\right)$
$\mathrm{R}=.03376$ or $3.376 \%$
Which means the YTM is:
$\mathrm{YTM}=3.376 \% \times 2$
$\mathrm{YTM}=6.75 \%$
So, the aftertax cost of the zero coupon bonds is:
$\mathrm{R}_{\mathrm{Z}}=.0675(1-.35)$
$\mathrm{R}_{\mathrm{Z}}=.0439$ or $4.39 \%$
The aftertax cost of debt for the company is the weighted average of the aftertax cost of debt for all outstanding bond issues. We need to use the market value weights of the bonds. The total aftertax cost of debt for the company is:
$\mathrm{R}_{\mathrm{D}}=.0473[(1.08)(\$ 60,000,000) / \$ 83,350,000]+.0439[(.265)(\$ 70,000,000) / \$ 83,350,000]$
$\mathrm{R}_{\mathrm{D}}=.0465$ or $4.65 \%$
9. $a$. Using the equation to calculate the WACC, we find:

```
\(\mathrm{WACC}=.70(.14)+.05(.06)+.25(.075)(1-.35)\)
\(\mathrm{WACC}=.1132\) or \(11.32 \%\)
```

b. Since interest is tax deductible and dividends are not, we must look at the aftertax cost of debt, which is:
$\mathrm{R}_{\mathrm{D}}=.075(1-.35)$
$R_{D}=.0488$ or $4.88 \%$
Hence, on an aftertax basis, debt is cheaper than the preferred stock.
10. Here, we need to use the debt-equity ratio to calculate the WACC. A debt-equity ratio of 6 implies a weight of debt of $.6 / 1.6$ and an equity weight of $1 / 1.6$. Using this relationship, we find:

WACC $=.14(1 / 1.40)+.08(.40 / 1.40)(1-.35)$
$\mathrm{WACC}=.1149$ or $11.49 \%$

## B-226 SOLUTIONS

11. Here, we have the WACC and need to find the debt-equity ratio of the company. Setting up the WACC equation, we find:
$\mathrm{WACC}=.105=.14(\mathrm{E} / \mathrm{V})+.08(\mathrm{D} / \mathrm{V})(1-.35)$
Rearranging the equation, we find:
$.105(\mathrm{~V} / \mathrm{E})=.14+.08(.65)(\mathrm{D} / \mathrm{E})$
Now we must realize that the $\mathrm{V} / \mathrm{E}$ is just the equity multiplier, which is equal to:
$\mathrm{V} / \mathrm{E}=1+\mathrm{D} / \mathrm{E}$
$.105(\mathrm{D} / \mathrm{E}+1)=.14+.052(\mathrm{D} / \mathrm{E})$
Now, we can solve for D/E as:
$.053(\mathrm{D} / \mathrm{E})=.0250$
D/E = . 6604
12. $a$. The book value of equity is the book value per share times the number of shares, and the book value of debt is the face value of the company's debt, so:
$B V_{E}=10,000,000(\$ 1)=\$ 10,000,000$
$B V_{D}=\$ 75,000,000+40,000,000=\$ 115,000,000$
So, the total value of the company is:
$\mathrm{V}=\$ 10,000,000+115,000,000=\$ 125,000,000$
And the book value weights of equity and debt are:
$\mathrm{E} / \mathrm{V}=\$ 10,000,000 / \$ 125,000,000=.08$
$\mathrm{D} / \mathrm{V}=1-\mathrm{E} / \mathrm{V}=.92$
b. The market value of equity is the share price times the number of shares, so:
$\mathrm{MV}_{\mathrm{E}}=10,000,000(\$ 53)=\$ 530,000,000$
Using the relationship that the total market value of debt is the price quote times the par value of the bond, we find the market value of debt is:
$\mathrm{MV}_{\mathrm{D}}=0.97(\$ 75,000,000)+0.96(\$ 40,000,000)=\$ 111,150,000$

This makes the total market value of the company:
$\mathrm{V}=\$ 530,000,000+111,500,000=\$ 641,150,000$
And the market value weights of equity and debt are:
$\mathrm{E} / \mathrm{V}=\$ 530,000,000 / \$ 641,150,000=.8266$
$\mathrm{D} / \mathrm{V}=1-\mathrm{E} / \mathrm{V}=.1734$
c. The market value weights are more relevant because they represent a more current valuation of the debt and equity.
13. First, we will find the cost of equity for the company. The information provided allows us to solve for the cost of equity using the dividend growth model, so:
$\mathrm{R}_{\mathrm{E}}=[\$ 2.60(1.07) / \$ 53]+.07$
$\mathrm{R}_{\mathrm{E}}=.1225$ or $12.25 \%$
Next, we need to find the YTM on both bond issues. Doing so, we find:
$\mathrm{P}_{1}=\$ 970=\$ 37.50\left(\mathrm{PVIFA}_{\mathrm{R} \%, 40}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 40}\right)$
R $=3.899 \%$
$\mathrm{YTM}=3.899 \% \times 2$
$\mathrm{YTM}=7.80 \%$
$\mathrm{P}_{2}=\$ 960=\$ 35\left(\mathrm{PVIFA}_{\mathrm{R} \%, 24}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 24}\right)$
$\mathrm{R}=3.756 \%$
$\mathrm{YTM}=3.756 \% \times 2$
$\mathrm{YTM}=7.51 \%$
To find the weighted average aftertax cost of debt, we need the weight of each bond as a percentage of the total debt. We find:
$\mathrm{w}_{\mathrm{D} 1}=.97(\$ 75,000,000) / \$ 111,150,000$
$\mathrm{w}_{\mathrm{D} 1}=.6545$
$\mathrm{w}_{\mathrm{D} 2}=.96(\$ 40,000,000) / \$ 111,150,000$
$\mathrm{w}_{\mathrm{D} 2}=.3455$
Now we can multiply the weighted average cost of debt times one minus the tax rate to find the weighted average aftertax cost of debt. This gives us:
$\mathrm{R}_{\mathrm{D}}=(1-.35)[(.6545)(.0780)+(.3455)(.0751)]$
$\mathrm{R}_{\mathrm{D}}=.0500$ or $5.00 \%$

## B-228 SOLUTIONS

Using these costs we have found and the weight of debt we calculated earlier, the WACC is:
$\mathrm{WACC}=.8266(.1225)+.1734(.0500)$
$\mathrm{WACC}=.1099$ or $10.99 \%$
14. a. Using the equation to calculate WACC, we find:

$$
\begin{aligned}
& \mathrm{WACC}=.105=(1 / 1.55)(.14)+(.55 / 1.55)(1-.35) \mathrm{R}_{\mathrm{D}} \\
& \mathrm{R}_{\mathrm{D}}=.0636 \text { or } 6.36 \%
\end{aligned}
$$

b. Using the equation to calculate WACC, we find:

$$
\begin{aligned}
& \mathrm{WACC}=.105=(1 / 1.55) \mathrm{R}_{\mathrm{E}}+(.55 / 1.55)(.065) \\
& \mathrm{R}_{\mathrm{E}}=.1270 \text { or } 12.70 \%
\end{aligned}
$$

15. We will begin by finding the market value of each type of financing. We find:

$$
\begin{aligned}
& \mathrm{MV}_{\mathrm{D}}=6,500(\$ 1,000)(1.04)=\$ 6,760,000 \\
& \mathrm{MV}_{\mathrm{E}}=150,000(\$ 78)=\$ 11,700,000 \\
& \mathrm{MV}_{\mathrm{P}}=10,000(\$ 80)=\$ 800,000
\end{aligned}
$$

And the total market value of the firm is:
$\mathrm{V}=\$ 6,760,000+11,700,000+800,000$
$\mathrm{~V}=\$ 19,260,000$
Now, we can find the cost of equity using the CAPM. The cost of equity is:
$R_{E}=.0525+1.15(.08)$
$R_{E}=.1445$ or $14.45 \%$
The cost of debt is the YTM of the bonds, so:

```
\(\mathrm{P}_{0}=\$ 1,040=\$ 42.50\left(\mathrm{PVIFA}_{\mathrm{R} \%, 50}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 50}\right)\)
R \(=4.062 \%\)
\(\mathrm{YTM}=4.062 \% \times 2\)
\(\mathrm{YTM}=8.12 \%\)
```

And the aftertax cost of debt is:
$\mathrm{R}_{\mathrm{D}}=(1-.35)(.0812)$
$\mathrm{R}_{\mathrm{D}}=.0528$ or $5.28 \%$
The cost of preferred stock is:
$\mathrm{R}_{\mathrm{P}}=\$ 6.25 / \$ 80$
$\mathrm{R}_{\mathrm{P}}=.0781$ or $7.81 \%$

Now we have all of the components to calculate the WACC. The WACC is:
WACC $=.0528(\$ 6,760 / \$ 19,260)+.1445(\$ 11,700 / \$ 19,260)+.0781(\$ 800 / \$ 19,260)$
$\mathrm{WACC}=.1096$ or $10.96 \%$
Notice that we didn't include the $\left(1-t_{\mathrm{C}}\right)$ term in the WACC equation. We used the aftertax cost of debt in the equation, so the term is not needed here.
16. $a$. We will begin by finding the market value of each type of financing. We find:

$$
\begin{aligned}
& \mathrm{MV}_{\mathrm{D}}=200,000(\$ 1,000)(1.08)=\$ 216,000,000 \\
& \mathrm{MV}_{\mathrm{E}}=9,000,000(\$ 64)=\$ 576,000,000 \\
& \mathrm{MV}_{\mathrm{P}}=500,000(\$ 83)=\$ 41,500,000
\end{aligned}
$$

And the total market value of the firm is:

$$
\begin{aligned}
& V=\$ 216,000,000+576,000,000+41,500,000 \\
& V=\$ 833,500,000
\end{aligned}
$$

So, the market value weights of the company's financing is:
$\mathrm{D} / \mathrm{V}=\$ 216,000,000 / \$ 833,500,000=.2591$
$\mathrm{P} / \mathrm{V}=\$ 41,500,000 / \$ 833,500,000=.0498$
$\mathrm{E} / \mathrm{V}=\$ 576,000,000 / \$ 833,500,000=.6911$
b. For projects equally as risky as the firm itself, the WACC should be used as the discount rate.

First, we can find the cost of equity using the CAPM. The cost of equity is:
$\mathrm{R}_{\mathrm{E}}=.055+1.10(.08)$
$\mathrm{R}_{\mathrm{E}}=.1430$ or $14.30 \%$
The cost of debt is the YTM of the bonds, so:
$\mathrm{P}_{0}=\$ 1,080=\$ 47\left(\mathrm{PVIFA}_{\mathrm{R} \%, 30}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 30}\right)$
R $=4.225 \%$
YTM $=4.225 \% \times 2$
$\mathrm{YTM}=8.45 \%$
And the aftertax cost of debt is:
$\mathrm{R}_{\mathrm{D}}=(1-.35)(.0845)$
$\mathrm{R}_{\mathrm{D}}=.0558$ or $5.58 \%$
The cost of preferred stock is:
$\mathrm{R}_{\mathrm{P}}=\$ 6 / \$ 83$
$\mathrm{R}_{\mathrm{P}}=.0723$ or $7.23 \%$

Now, we can calculate the WACC as:

$$
\begin{aligned}
& \mathrm{WACC}=.2591(.0558)+.0498(.0723)+.6911(.1430) \\
& \mathrm{WACC}=.1169 \text { or } 11.69 \%
\end{aligned}
$$

17. a. Projects Y and Z .
b. Using the CAPM to consider the projects, we need to calculate the expected return of the project, given its level of risk. This expected return should then be compared to the expected return of the project. If the return calculated using the CAPM is higher than the project expected return, we should accept the project; if not, we reject the project. After considering risk via the CAPM:

$$
\begin{aligned}
& \mathrm{E}[\mathrm{~W}]=.05+.70(.13-.05)=.1060>.10 \text {, so reject } \mathrm{W} \\
& \mathrm{E}[\mathrm{X}]=.05+.90(.13-.05)=.1220<.125 \text {, so accept } \mathrm{X} \\
& \mathrm{E}[\mathrm{Y}]=.05+1.20(.13-.05)=.1460>.14 \text {, so reject } \mathrm{Y} \\
& \mathrm{E}[\mathrm{Z}]=.05+1.80(.13-.05)=.1940<.21 \text {, so accept } \mathrm{Z}
\end{aligned}
$$

c. Project X would be incorrectly rejected; Project Y would be incorrectly accepted.
18. We will begin by finding the market value of each type of financing. We find:

$$
\begin{aligned}
& \mathrm{MV}_{\mathrm{D}}=7,000(\$ 1,000)(1.08)=\$ 7,560,000 \\
& \mathrm{MV}_{\mathrm{E}}=180,000(\$ 60)=\$ 10,800,000 \\
& \mathrm{MV}_{\mathrm{P}}=8,000(\$ 94)=\$ 752,000
\end{aligned}
$$

And the total market value of the firm is:
$V=\$ 7,560,000+10,800,000+756,000$
$\mathrm{V}=\$ 19,112,000$
Now, we can find the cost of equity using the CAPM. The cost of equity is:
$\mathrm{R}_{\mathrm{El}}=.05+.90(.12-.05)$
$\mathrm{R}_{\mathrm{El}}=.1130$ or $11.30 \%$
We can also find the cost of equity, using the dividend discount model. The cost of equity with the dividend discount model is:
$\mathrm{R}_{\mathrm{E} 2}=(\$ 2.80 / \$ 60)+.06$
$\mathrm{R}_{\mathrm{E} 2}=.1067$ or $10.67 \%$
Both estimates for the cost of equity seem reasonable, so we will use the average of the two. The cost of equity estimate is:
$\mathrm{R}_{\mathrm{E}}=(.1130+.1067) / 2$
$\mathrm{R}_{\mathrm{E}}=.1098$ or $10.98 \%$

The cost of debt is the YTM of the bonds, so:
$\mathrm{P}_{0}=\$ 1,080=\$ 37.25\left(\mathrm{PVIFA}_{\mathrm{R} \%, 40}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 40}\right)$
$\mathrm{R}=3.382 \%$
YTM $=3.382 \% \times 2$
$\mathrm{YTM}=6.76 \%$
And the aftertax cost of debt is:
$R_{D}=(1-.35)(.0676)$
$R_{D}=.0440$ or $4.40 \%$
The cost of preferred stock is:
$\mathrm{R}_{\mathrm{P}}=\$ 5.50 / \$ 94$
$\mathrm{R}_{\mathrm{P}}=.0585$ or $5.85 \%$
Now, we have all of the components to calculate the WACC. The WACC is:
$\mathrm{WACC}=.0440(\$ 7,560 / \$ 19,112)+.0585(\$ 752 / \$ 19,112)+.1098(\$ 10,800 / \$ 19,112)$
WACC $=.0818$ or $8.18 \%$
19. The bonds have 26 years to maturity so the price today is:
$\mathrm{P}_{0}=\$ 1,000 /(1+.073 / 2)^{52}$
$\mathrm{P}_{0}=\$ 155.02$
The market value of the debt is:
$M V_{D}=90,000(\$ 155.02)=\$ 13,952,067.70$
So, the total value of the firm is:
$V=\$ 13,952,067.70+40,000,000$
$\mathrm{V}=\$ 53,952,067.70$
This means the weight of debt in the capital structure is:
D/V $=\$ 13,952,067.70 / \$ 53,952,067.70$
$\mathrm{D} / \mathrm{V}=.2586$
20. To find the required return for the project, we need to adjust the company's WACC for the level of risk in the project. A debt-equity ratio of .40 implies a weight of debt of $.40 / 1.40$ and a weight of equity of $1 / 1.40$, so the company's WACC is:
$\mathrm{WACC}=(.40 / 1.40)(.0625)+(1 / 1.40)(.1350)$
$\mathrm{WACC}=.1143$ or $11.43 \%$

## B-232 SOLUTIONS

Adjusting for risk, the project discount rate is:
Project discount rate $=.1143+.03$
Project discount rate $=.1443$ or $14.43 \%$

## Intermediate

21. First, we need to find the project discount rate. The project discount rate is the company's cost of capital plus a risk adjustment factor. A debt-equity ratio of .50 implies a weight of debt of $.50 / 1.50$ and a weight of equity of $1 / 1.50$, so the company's WACC is:
$\mathrm{WACC}=(.50 / 1.50)(.07)+(1 / 1.50)(.14)$
$\mathrm{WACC}=.1167$ or $11.67 \%$

Adjusting for risk, the project discount rate is:

Project discount rate $=.1167+.02$
Project discount rate $=.1367$ or $13.67 \%$

The company should only accept the project if the NPV is zero (hopefully greater than zero.) The cash flows are a growing annuity. The present value of a growing annuity can be found with the dividend discount equation. So, the present value of the savings is:
$\mathrm{PV}=[\$ 4,500,000 /(.1367-.04)]$
PV $=\$ 46,551,724.14$

The project should only be undertaken if its cost is less than $\$ 46,551,724.14$.
22. To find the aftertax cost of equity for the company, we need to find the weighted average of the four debt issues. We will begin by calculating the market value of each debt issue, which is:

```
MV 
MV =$22,000,000
MV 
MV = $42,400,000
MV = 1.09($45,000,000)
MV 3 = $49,050,000
MV 
MV 4 = $67,200,000
```

So, the total market value of the company's debt is:
$M V_{D}=\$ 22,000,000+42,400,000+49,050,000+67,200,000$
$M V_{D}=\$ 180,650,000$

The weight of each debt issue is:

```
\(\mathrm{w}_{1}=\$ 22,000,000 / \$ 180,650,000=.1218\)
\(\mathrm{w}_{1}=.1218\) or \(12.18 \%\)
\(\mathrm{w}_{2}=\$ 44,000,000 / \$ 180,650,000\)
\(\mathrm{w}_{2}=.2347\) or \(23.47 \%\)
\(\mathrm{w}_{3}=\$ 49,050,000 / \$ 180,650,000\)
\(\mathrm{w}_{3}=.2715\) or \(27.15 \%\)
\(\mathrm{w}_{4}=\$ 67,200,000 / \$ 180,650,000\)
\(\mathrm{w}_{4}=.3720\) or \(37.20 \%\)
```

Next, we need to find the YTM for each bond issue. The YTM for each issue is:
$\mathrm{P}_{1}=\$ 1,100=\$ 41\left(\mathrm{PVIFA}_{\mathrm{R} \%, 10}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 10}\right)$
$\mathrm{R}_{1}=.02932$ or $2.932 \%$
$\mathrm{YTM}_{1}=2.932 \% \times 2$
$\mathrm{YTM}_{1}=5.86 \%$
$\mathrm{P}_{2}=\$ 1,060=\$ 37\left(\mathrm{PVIFA}_{\mathrm{R} \%, 16}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 16}\right)$
$\mathrm{R}_{2}=.03214$ or $3.214 \%$
$\mathrm{YTM}_{2}=3.214 \% \times 2$
$\mathrm{YTM}_{2}=6.43 \%$
$\mathrm{P}_{3}=\$ 1,090=\$ 40\left(\mathrm{PVIFA}_{\mathrm{R} \%, 31}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 31}\right)$
$\mathrm{R}_{3}=.03519$ or $3.519 \%$
$\mathrm{YTM}_{3}=3.519 \% \times 2$
$\mathrm{YTM}_{3}=7.04 \%$
$\mathrm{P}_{4}=\$ 1,120=\$ 42\left(\mathrm{PVIFA}_{\mathrm{R} \%, 50}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 50}\right)$
$\mathrm{R}_{4}=.03672$ or $3.672 \%$
$\mathrm{YTM}_{4}=3.672 \% \times 2$
$\mathrm{YTM}_{4}=7.34 \%$
The weighted average YTM of the company's debt is thus:
$\mathrm{YTM}=.1218(.0387)+.2347(.0424)+.2715(.0704)+.3720(.0734)$
$\mathrm{YTM}=.0687$ or $6.87 \%$
And the aftertax cost of debt is:
$\mathrm{R}_{\mathrm{D}}=.0687(1-.034)$
$\mathrm{R}_{\mathrm{D}}=.0453$ or $4.52 \%$

## B-234 SOLUTIONS

23. $a$. Using the dividend discount model, the cost of equity is:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{E}}=[(0.80)(1.06) / \$ 72]+.06 \\
& \mathrm{R}_{\mathrm{E}}=.0718 \text { or } 7.18 \%
\end{aligned}
$$

b. Using the CAPM, the cost of equity is:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{E}}=.05+1.20(.1250-.05) \\
& \mathrm{R}_{\mathrm{E}}=.1400 \text { or } 14.00 \%
\end{aligned}
$$

c. When using the dividend growth model or the CAPM, you must remember that both are estimates for the cost of equity. Additionally, and perhaps more importantly, each method of estimating the cost of equity depends upon different assumptions.

## Challenge

24. We can use the debt-equity ratio to calculate the weights of equity and debt. The debt of the company has a weight for long-term debt and a weight for accounts payable. We can use the weight given for accounts payable to calculate the weight of accounts payable and the weight of long-term debt. The weight of each will be:

Accounts payable weight $=.20 / 1.20=.17$
Long-term debt weight $=1 / 1.20=.83$
Since the accounts payable has the same cost as the overall WACC, we can write the equation for the WACC as:
$\mathrm{WACC}=(1 / 1.8)(.17)+(0.8 / 1.8)[(.20 / 1.2) \mathrm{WACC}+(1 / 1.2)(.09)(1-.35)]$
Solving for WACC, we find:
WACC $=.0944+.4444[(.20 / 1.2) \mathrm{WACC}+.0488]$
$\mathrm{WACC}=.0944+(.0741) \mathrm{WACC}+.0217$
(.9259)WACC = . 1161
$\mathrm{WACC}=.1254$ or $12.54 \%$
Since the cash flows go to perpetuity, we can calculate the present value using the equation for the PV of a perpetuity. The NPV is:
$\mathrm{NPV}=-\$ 80,000,000+(\$ 10,900,000 / .1254)$
NPV = \$6,921,850
25. a. The $\$ 6$ million cost of the land 3 years ago is a sunk cost and irrelevant; the $\$ 6.4$ million appraised value of the land is an opportunity cost and is relevant. So, the total initial cash flow is:

$$
\begin{aligned}
& \mathrm{CF}_{0}=-\$ 6,400,000-9,800,000-825,000 \\
& \mathrm{CF}_{0}=-\$ 17,025,000
\end{aligned}
$$

b. To find the required return for the project, we need to adjust the company's WACC for the level of risk in the project. We begin by calculating the market value of each type of financing, so:
$\mathrm{MV}_{\mathrm{D}}=25,000(\$ 1,000)(0.96)=\$ 24,000,000$
$\mathrm{MV}_{\mathrm{E}}=400,000(\$ 89)=\$ 35,600,000$
$\mathrm{MV}_{\mathrm{P}}=35,000(\$ 99)=\$ 3,465,000$
The total market value of the company is:
$V=\$ 24,000,000+35,600,000+3,465,000$
$\mathrm{V}=\$ 63,065,000$
Next, we need to find the cost of funds. We have the information available to calculate the cost of equity, using the CAPM, so:
$\mathrm{R}_{\mathrm{E}}=.0520+1.20(.08)$
$\mathrm{R}_{\mathrm{E}}=.1480$ or $14.80 \%$
The cost of debt is the YTM of the company's outstanding bonds, so:
$\mathrm{P}_{0}=\$ 960=\$ 32.50\left(\mathrm{PVIFA}_{\mathrm{R} \%, 40}\right)+\$ 1,000\left(\mathrm{PVIF}_{\mathrm{R} \%, 40}\right)$
$\mathrm{R}=.03435$ or $3.435 \%$
$\mathrm{YTM}=3.435 \% \times 2$
$\mathrm{YTM}=6.87 \%$
And the aftertax cost of debt is:
$\mathrm{R}_{\mathrm{D}}=(1-.34)(.0687)$
$\mathrm{R}_{\mathrm{D}}=.0453$ or $4.53 \%$
The cost of preferred stock is:
$\mathrm{R}_{\mathrm{P}}=\$ 6.5 / \$ 99$
$\mathrm{R}_{\mathrm{P}}=.0657$ or $6.57 \%$
So, the company's WACC is:
WACC $=.0453(\$ 24,000 / \$ 63,065)+.0657(\$ 3,465 / \$ 63,035)+.1480(\$ 35,600 / \$ 63,065)$
$W A C C=.1044$ or $10.44 \%$
The company wants to use the subjective approach to this project because it is located overseas. The adjustment factor is 2 percent, so the required return on this project is:

Project required return $=.1044+.02$
Project required return $=.1244$ or $12.44 \%$

## B-236 SOLUTIONS

c. The annual depreciation for the equipment will be:
\$9,800,000/8 = \$1,225,000
So, the book value of the equipment at the end of five years will be:
$\mathrm{BV}_{5}=\$ 9,800,000-5(\$ 1,225,000)$
$\mathrm{BV}_{5}=\$ 3,675,000$
So, the aftertax salvage value will be:
Aftertax salvage value $=\$ 1,250,000+.34(\$ 3,675,000-1,250,000)$
Aftertax salvage value $=\$ 2,074,500$
d. Using the tax shield approach, the OCF for this project is:

```
\(\mathrm{OCF}=[(\mathrm{P}-\mathrm{v}) \mathrm{Q}-\mathrm{FC}](1-\mathrm{t})+\mathrm{t}_{\mathrm{C}} \mathrm{D}\)
OCF \(=[(\$ 10,000-9,300)(11,000)-2,100,000](1-.34)+.34(\$ 9,800,000 / 8)\)
\(\mathrm{OCF}=\$ 4,112,500\)
```

e. We have calculated all cash flows of the project. We just need to make sure that in Year 5 we add back the aftertax salvage value, the recovery of the initial NWC, and the aftertax value of the land in five years since it will be an opportunity cost. So, the cash flows for the project are:

| Year | Flow Cash |
| :---: | :---: |
| 0 | -\$17,025,000 |
| 1 | 4,112,500 |
| 2 | 4,112,500 |
| 3 | 4,112,500 |
| 4 | 4,112,500 |
| 5 | 14,012,000 |

Using the required return of 12.71 percent, the NPV of the project is:
$\mathrm{NPV}=-\$ 17,025,000+\$ 4,112,500\left(\mathrm{PVIFA}_{12.44 \%, 4}\right)+\$ 14,012,000 / 1.1244^{5}$
$\mathrm{NPV}=\$ 3,147,020.42$
And the IRR is:

```
\(\mathrm{NPV}=0=-\$ 17,025,000+\$ 4,112,500\left(\mathrm{PVIFA}_{\text {IRR }} \%, 4\right)+\$ 14,012,000 /(1+\mathrm{IRR})^{5}\)
IRR \(=18.35 \%\)
```


## CHAPTER 13 <br> LEVERAGE AND CAPITAL STRUCTURE

## Answers to Concepts Review and Critical Thinking Questions

1. Business risk is the equity risk arising from the nature of the firm's operating activity, and is directly related to the systematic risk of the firm's assets. Financial risk is the equity risk that is due entirely to the firm's chosen capital structure. As financial leverage, or the use of debt financing, increases, so does financial risk and hence the overall risk of the equity. Thus, Firm B could have a higher cost of equity if it uses greater leverage.
2. No, it doesn't follow. While it is true that the equity and debt costs are rising, the key thing to remember is that the cost of debt is still less than the cost of equity. Since we are using more and more debt, the WACC does not necessarily rise.
3. Because many relevant factors such as bankruptcy costs, tax asymmetries, and agency costs cannot easily be identified or quantified, it's practically impossible to determine the precise debt/equity ratio that maximizes the value of the firm. However, if the firm's cost of new debt suddenly becomes much more expensive, it's probably true that the firm is too highly leveraged.
4. The more capital intensive industries, such as airlines, cable television, and electric utilities, tend to use greater financial leverage. Also, industries with less predictable future earnings, such computers or drugs, tend to use less. Such industries also have a higher concentration of growth and startup firms. Overall, the general tendency is for firms with identifiable, tangible assets and relatively more predictable future earnings to use more debt financing. These are typically the firms with the greatest need for external financing and the greatest likelihood of benefiting from the interest tax shelter.
5. It's called leverage (or "gearing" in the UK) because it magnifies gains or losses.
6. Homemade leverage refers to the use of borrowing on the personal level as opposed to the corporate level.
7. One answer is that the right to file for bankruptcy is a valuable asset, and the financial manager acts in shareholders' best interest by managing this asset in ways that maximize its value. To the extent that a bankruptcy filing prevents "a race to the courthouse steps," it would seem to be a reasonable alternative to complicated and expensive litigation.
8. As in the previous question, it could be argued that using bankruptcy laws as a sword may simply be the best use of the asset. Creditors are aware at the time a loan is made of the possibility of bankruptcy, and the interest charged incorporates this possibility.

## B-238 SOLUTIONS

9. One side is that Continental was going to go bankrupt because its costs made it uncompetitive. The bankruptcy filing enabled Continental to restructure and keep flying. The other side is that Continental abused the bankruptcy code. Rather than renegotiate labor agreements, Continental simply abrogated them to the detriment of its employees. It is important thing to keep in mind that the bankruptcy code is a creation of law, not economics. A strong argument can always be made that making the best use of the bankruptcy code is no different from, for example, minimizing taxes by making best use of the tax code. Indeed, a strong case can be made that it is the financial manager's duty to do so. As the case of Continental illustrates, the code can be changed if socially undesirable outcomes are a problem.
10. As with any management decision, the goal is to maximize the value of shareholder equity. To accomplish this with respect to the capital structure decision, management attempts to choose the capital structure with the lowest cost of capital.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. a. A table outlining the income statement for the three possible states of the economy is shown below. The EPS is the net income divided by the 3,500 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

| EBIT | Recession <br> Interest | $\$ 3,600$ | Normal <br> $\$ 6,000$ |
| :--- | ---: | ---: | ---: |
| NI | $\underline{0}$ | $\underline{0}$ | Expansion <br> $\$ 7,500$ <br> EPS |
| $\$ 3,600$ | $\$ 6,000$ | $\$ 7,500$ |  |
| $\% \Delta$ EPS | $\$ 1.03$ | $\$ 1.71$ | $\$ 2.14$ |
|  | $-40 \%$ | - | $+25 \%$ |

b. If the company undergoes the proposed recapitalization, it will repurchase:

Share price $=$ Equity $/$ Shares outstanding
Share price $=\$ 70,000 / 3,500$
Share price $=\$ 20$
Shares repurchased $=$ Debt issued $/$ Share price
Shares repurchased $=\$ 35,000 / \$ 20$
Shares repurchased $=1,750$

The interest payment each year under all three scenarios will be:
Interest payment $=\$ 35,000(.06)$
Interest payment $=\$ 2,100$
The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy under the proposed recapitalization.

|  | Recession | Normal | Expansion |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 3,600$ | $\$ 6,000$ | $\$ 7,500$ |
| Interest | $\underline{2,100}$ | $\underline{2,100}$ | $\underline{2,100}$ |
| NI | $\underline{\$ 1,500}$ | $\underline{\underline{\$ 3,900}}$ | $\underline{\underline{\$ 5,400}}$ |
| EPS | $-61.54 \%$ | $\$ 2.23$ | $\$ 3.09$ |
| $\% \Delta$ EPS | - | $+38.46 \%$ |  |

2. a. A table outlining the income statement with taxes for the three possible states of the economy is shown below. The share price is still $\$ 20$, and there are still 3,500 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

|  | Recession | Normal | Expansion |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 3,600$ | $\$ 6,000$ | $\$ 7,500$ |
| Interest | 0 | 0 | 0 |
| Taxes | $\underline{1,260}$ | $\underline{2,100}$ | $\underline{2,625}$ |
| NI | $\underline{\$ 2,340}$ | $\underline{\$ 3,900}$ | $\underline{\underline{\$ 4,875}}$ |
| EPS | $\$ 0.67$ | $\$ 1.11$ | $\$ 1.39$ |
| $\% \Delta$ EPS | $-40 \%$ | - | +25 |

b. A table outlining the income statement with taxes for the three possible states of the economy and assuming the company undertakes the proposed capitalization is shown below. The interest payment and shares repurchased are the same as in part $b$ of Problem 1.

|  | Recession | Normal | Expansion |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 3,600$ | $\$ 6,000$ | $\$ 7,500$ |
| Interest | 2,100 | 2,100 | 2,100 |
| Taxes | $\underline{525}$ | $\underline{1,365}$ | $\underline{1,890}$ |
| NI | $\underline{\$ 975}$ | $\underline{\$ 2,535}$ | $\underline{\$ 3,510}$ |
| EPS | $\$ 0.56$ | $\$ 1.45$ | $\$ 2.01$ |
| $\% \Delta$ EPS | $-61.54 \%$ | - | $+38.46 \%$ |

Notice that the percentage change in EPS is the same both with and without taxes

## B-240 SOLUTIONS

3. $a$. Since the company has a market-to-book ratio of 1.0 , the total equity of the firm is equal to the market value of equity. Using the equation for ROE:

ROE $=\mathrm{NI} / \$ 70,000$
The ROE for each state of the economy under the current capital structure and no taxes is:

|  | Recession | Normal | Expansion |
| :--- | ---: | ---: | ---: |
| ROE | $5.14 \%$ | $8.57 \%$ | $10.71 \%$ |
| $\% \triangle$ ROE | $-40 \%$ | - | $+25 \%$ |

The second row shows the percentage change in ROE from the normal economy.
$b$. If the company undertakes the proposed recapitalization, the new equity value will be:
Equity $=\$ 70,000-35,000$
Equity $=\$ 35,000$
So, the ROE for each state of the economy is:
ROE $=\mathrm{NI} / \$ 35,000$

|  | Recession | Normal | Expansion |
| :--- | ---: | ---: | ---: |
| ROE | $4.29 \%$ | $11.14 \%$ | $15.43 \%$ |
| $\% \triangle$ ROE | $-61.54 \%$ | - | $+38.46 \%$ |

c. If there are corporate taxes and the company maintains its current capital structure, the ROE is:

| ROE | $3.34 \%$ | $5.57 \%$ | $6.96 \%$ |
| :--- | ---: | ---: | ---: |
| $\% \triangle$ ROE | $-40 \%$ | - | $+25 \%$ |

If the company undertakes the proposed recapitalization, and there are corporate taxes, the ROE for each state of the economy is:

| ROE | $2.79 \%$ | $7.24 \%$ | $10.03 \%$ |
| :--- | ---: | ---: | ---: |
| $\% \triangle$ ROE | $-61.54 \%$ | - | $+38.46 \%$ |

Notice that the percentage change in ROE is the same as the percentage change in EPS. The percentage change in ROE is also the same with or without taxes.
4. a. Under Plan I, the unlevered company, net income is the same as EBIT with no corporate tax. The EPS under this capitalization will be:

EPS $=\$ 1,500,000 / 900,000$ shares
EPS $=\$ 1.67$

Under Plan II, the levered company, net income will be reduced by the interest payment. The interest payment is the amount of debt times the interest rate, so:

$$
\begin{aligned}
& \mathrm{NI}=\$ 1,500,000-.10(\$ 10,000,000) \\
& \mathrm{NI}=\$ 500,000
\end{aligned}
$$

And the EPS will be:

$$
\begin{aligned}
& \mathrm{EPS}=\$ 500,000 / 650,000 \text { shares } \\
& \mathrm{EPS}=\$ 0.77
\end{aligned}
$$

Plan I has the higher EPS when EBIT is $\$ 1,500,000$.
b. Under Plan I, the net income is $\$ 5,000,000$ and the EPS is:
$\mathrm{EPS}=\$ 5,000,000 / 900,000$ shares
$\mathrm{EPS}=\$ 5.56$
Under Plan II, the net income is:

$$
\begin{aligned}
& \mathrm{NI}=\$ 5,000,000-.10(\$ 10,000,000) \\
& \mathrm{NI}=\$ 4,000,000
\end{aligned}
$$

And the EPS is:

$$
\begin{aligned}
& \mathrm{EPS}=\$ 4,000,000 / 650,000 \text { shares } \\
& \mathrm{EPS}=\$ 6.15
\end{aligned}
$$

Plan II has the higher EPS when EBIT is $\$ 5,000,000$.
c. To find the breakeven EBIT for two different capital structures, we simply set the equations for EPS equal to each other and solve for EBIT. The breakeven EBIT is:

$$
\begin{aligned}
& \text { EBIT/ } 900,000=[\text { EBIT }-.10(\$ 10,000,000)] / 650,000 \\
& \text { EBIT }=\$ 3,600,000
\end{aligned}
$$

5. We can find the price per share by dividing the amount of debt used to repurchase shares by the number of shares repurchased. Doing so, we find the share price is:

Share price $=\$ 10,000,000 /(900,000-650,000)$
Share price $=\$ 40$ per share
The value of the company under the all-equity plan is:
$\mathrm{V}=\$ 40(900,000$ shares $)$
$\mathrm{V}=\$ 36,000,000$

## B-242 SOLUTIONS

And the value of the company under the levered plan is:
$\mathrm{V}=\$ 40(650,000$ shares $)+\$ 10,000,000$ debt
$\mathrm{V}=\$ 36,000,000$
6. a. The income statement for each capitalization plan is:

|  |  |  |  |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 15,000$ | $\$ 15,000$ | All-equity |
| Interest | $\underline{12,000}$ | $\underline{7,200}$ | $\frac{0}{0}$ |
| NI | $\underline{\$ 3,000}$ | $\underline{\$ 7,800}$ | $\underline{\$ 15,000}$ |
| EPS | $\$ 0.60$ | $\$ 1.11$ | $\$ 1.50$ |

Plan I has the lowest EPS; the all-equity plan has the highest EPS.
b. The breakeven level of EBIT occurs when the capitalization plans result in the same EPS. The EPS is calculated as:

EPS $=\left(E B I T-R_{D} D\right) /$ Shares outstanding
This equation calculates the interest payment $\left(\mathrm{R}_{\mathrm{D}} \mathrm{D}\right)$ and subtracts it from the EBIT, which results in the net income. Dividing by the shares outstanding gives us the EPS. For the all-equity capital structure, the interest term is zero. To find the breakeven EBIT for two different capital structures, we simply set the equations equal to each other and solve for EBIT. The breakeven EBIT between the all-equity capital structure and Plan I is:

EBIT/10,000 $=[$ EBIT $-.10(\$ 120,000)] / 5,000$
EBIT $=\$ 24,000$
And the breakeven EBIT between the all-equity capital structure and Plan II is:
EBIT $/ 10,000=[$ EBIT $-.10(\$ 72,000)] / 7,000$
EBIT $=\$ 24,000$
The break-even levels of EBIT are the same because of M\&M Proposition I.
c. Setting the equations for EPS from Plan I and Plan II equal to each other and solving for EBIT, we get:
$[$ EBIT $-.10(\$ 120,000)] / 5,000=[E B I T-.10(\$ 72,000)] / 7,000$
EBIT $=\$ 24,000$
This break-even level of EBIT is the same as in part $b$ again because of M\&M Proposition I.
d. The income statement for each capitalization plan with corporate income taxes is:

|  | $\underline{I}$ | $I I$ | All-equity |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 15,000$ | $\$ 15,000$ | $\$ 15,000$ |
| Interest | 12,000 | 7,200 | 0 |
| Taxes | $\underline{1,40}$ | $\underline{2,964}$ | $\underline{1,900}$ |
| NI | $\underline{\$ 1,860}$ | $\underline{\$ 4,836}$ | $\underline{\$ 9,300}$ |
| EPS | $\$ 0.37$ | $\$ 0.69$ | $\$ 0.93$ |

The all-equity plan still has the highest EPS; Plan I still has the lowest EPS.
We can calculate the EPS as:
EPS $=\left[\left(E B I T-R_{D} D\right)\left(1-t_{C}\right)\right] /$ Shares outstanding
This is similar to the equation we used before, except now we need to account for taxes. Again, the interest expense term is zero in the all-equity capital structure. So, the breakeven EBIT between the all-equity plan and Plan I is:
$\operatorname{EBIT}(1-.38) / 10,000=[$ EBIT $-.10(\$ 120,000)](1-.38) / 5,000$
EBIT $=\$ 24,000$
The breakeven EBIT between the all-equity plan and Plan II is:
$\operatorname{EBIT}(1-.38) / 10,000=[E B I T-.10(\$ 72,000)](1-.38) / 7,000$
EBIT $=\$ 24,000$
And the breakeven between Plan I and Plan II is:
$[$ EBIT $-.10(\$ 120,000)](1-.38) / 5,000=[E B I T-.10(\$ 72,000)](1-.38) / 5,000$
EBIT $=\$ 24,000$
The break-even levels of EBIT do not change because the addition of taxes reduces the income of all three plans by the same percentage; therefore, they do not change relative to one another.
7. To find the value per share of the stock under each capitalization plan, we can calculate the price as the value of shares repurchased divided by the number of shares repurchased. So, under Plan I, the value per share is:
$\mathrm{P}=\$ 120,000 / 5,000$ shares
$\mathrm{P}=\$ 24$ per share

## B-244 SOLUTIONS

And under Plan II, the value per share is:
$\mathrm{P}=\$ 72,000 / 3,000$ shares
$\mathrm{P}=\$ 24$ per share
This shows that when there are no corporate taxes, the stockholder does not care about the capital structure decision of the firm. This is M\&M Proposition I without taxes.
8. $a$. The earnings per share are:

```
EPS \(=\$ 17,600 / 5,500\) shares
EPS \(=\$ 3.20\)
```

Since all earnings are paid as dividends, the cash flow for the investors will be:
Cash flow $=\$ 3.20$ ( 100 shares)
Cash flow $=\$ 320$
b. To determine the cash flow to the shareholder, we need to determine the EPS of the firm under the proposed capital structure. The market value of the firm is:
$\mathrm{V}=\$ 60(5,500)$
$\mathrm{V}=\$ 330,000$
Under the proposed capital structure, the firm will raise new debt in the amount of:
$\mathrm{D}=0.25(\$ 330,000)$
$\mathrm{D}=\$ 82,500$
This means the number of shares repurchased will be:
Shares repurchased $=\$ 82,500 / \$ 60$
Shares repurchased $=1,375$
Under the new capital structure, the company will have to make an interest payment on the new debt. The net income with the interest payment will be:
$\mathrm{NI}=\$ 17,600-.08(\$ 82,500)$
$\mathrm{NI}=\$ 11,000$
This means the EPS under the new capital structure will be:
EPS $=\$ 11,000 /(5,500-1,375)$
EPS $=\$ 2.67$

Since all earnings are paid as dividends, the shareholder will receive:
Shareholder cash flow $=\$ 2.67$ (100 shares)
Shareholder cash flow $=\$ 266.67$
c. To replicate the original capital structure, the shareholder should sell 25 percent of their shares, or 25 shares, and lend the proceeds at 8 percent. The shareholder will have an interest cash flow of:

Interest cash flow $=25(\$ 60)(.08)$
Interest cash flow $=\$ 120$
The shareholder will receive dividend payments on the remaining 75 shares, so the dividends received will be:

Dividends received $=\$ 2.67$ (75 shares)
Dividends received $=\$ 200$
The total cash flow for the shareholder under these assumptions will be:
Total cash flow $=\$ 120+200$
Total cash flow $=\$ 320$
This is the same cash flow we calculated in part $a$.
d. The capital structure is irrelevant because shareholders can create their own leverage or unlever the stock to create the payoff they desire, regardless of the capital structure the firm actually chooses.
9. $a$. The total value of the company is the share price times the number of shares, so:
$\mathrm{V}=\$ 80(3,000)$
$\mathrm{V}=\$ 240,000$
The investor will receive dividends in proportion to the percentage of the company's share they own. The total dividends received by the shareholder will be:

Dividends received $=\$ 34,500(\$ 20,000 / \$ 240,000)$
Dividends received $=\$ 2,875$
b. Under the proposed capital structure, the firm will raise new debt in the amount of:
$\mathrm{D}=0.30(\$ 240,000)$
$\mathrm{D}=\$ 72,000$

## B-246 SOLUTIONS

This means the number of shares repurchased will be:
Shares repurchased $=\$ 72,000 / \$ 80$
Shares repurchased $=900$
Under the new capital structure, the company will have to make an interest payment on the new debt. The net income with the interest payment will be:
$\mathrm{NI}=\$ 34,500-.07(\$ 72,000)$
$\mathrm{NI}=\$ 29,460$
This means the EPS under the new capital structure will be:
EPS $=\$ 29,640 /(3,000-900)$
EPS $=\$ 14.03$
The number of shares owned by the shareholder is the dollar amount invested divided by the share price, so:

Shares owned = \$20,000 / \$80
Shares owned $=250$
Since all earnings are paid as dividends, the shareholder will receive:
Shareholder cash flow $=\$ 14.03(250$ shares $)$
Shareholder cash flow $=\$ 3,507.14$
c. To replicate the proposed capital structure, the shareholder should sell 30 percent of their shares, or 75 shares, and lend the proceeds at 7 percent. The shareholder will have an interest cash flow of:

Interest cash flow $=75(\$ 80)(.07)$
Interest cash flow $=\$ 420$
The shareholder will receive dividend payments on the remaining 96 shares, so the dividends received will be:

Dividends received $=\$ 14.03$ ( 175 shares)
Dividends received $=\$ 2,455$
The total cash flow for the shareholder under these assumptions will be:
Total cash flow $=\$ 420+2,455$
Total cash flow $=\$ 2,875$
This is the same cash flow we calculated in part $a$.
d. The capital structure is irrelevant because shareholders can create their own leverage or unlever the stock to create the payoff they desire, regardless of the capital structure the firm actually chooses.
10. a. With the information provided, we can use the equation for calculating WACC to find the cost of equity. The equation for WACC (assuming no taxes) is:
$W A C C=(E / V) R_{E}+(D / V) R_{D}$
The company has a debt-equity ratio of 1.5 , which implies the weight of debt is $1.5 / 2.5$, and the weight of equity is $1 / 2.5$, so
$\mathrm{WACC}=.10=(1 / 2.5) \mathrm{R}_{\mathrm{E}}+(1.5 / 2.5)(.07)$
$\mathrm{R}_{\mathrm{E}}=.1450$ or $14.50 \%$
b. To find the cost of equity under different capital structures, we can again use the WACC equation. With a debt-equity ratio of 2 , the cost of equity is:
$.10=(1 / 3) \mathrm{R}_{\mathrm{E}}+(2 / 3)(.07)$
$\mathrm{R}_{\mathrm{E}}=.1600$ or $16.00 \%$
With a debt-equity ratio of 0.5 , the cost of equity is:
$.10=(1 / 1.5) \mathrm{R}_{\mathrm{E}}+(0.5 / 1.5)(.07)$
$\mathrm{R}_{\mathrm{E}}=.1150$ or $11.50 \%$
And with a debt-equity ratio of 0 , the cost of equity is:
$.10=(1) \mathrm{R}_{\mathrm{E}}+(0)(.07)$
$\mathrm{R}_{\mathrm{E}}=\mathrm{WACC}=.1000$ or $10.00 \%$
11. a. For an all-equity financed company:
$\mathrm{WACC}=\mathrm{R}_{\mathrm{E}}=.13$ or $13 \%$
b. To find the cost of equity for the company with leverage, we need to use M\&M Proposition I with no taxes, so:
$R_{E}=R_{A}+\left(R_{A}-R_{D}\right)(D / E)$
$\mathrm{R}_{\mathrm{E}}=.13+(.13-.08)(.30 / .70)$
$\mathrm{R}_{\mathrm{E}}=.1514$ or $15.14 \%$
c. Using M\&M Proposition I with no taxes again, we get:
$R_{E}=R_{A}+\left(R_{A}-R_{D}\right)(D / E)$
$\mathrm{R}_{\mathrm{E}}=.13+(.13-.08)(.60 / .40)$
$\mathrm{R}_{\mathrm{E}}=.2050$ or $20.50 \%$

## B-248 SOLUTIONS

d. The WACC with 30 percent debt is:

```
\(W A C C=(E / V) R_{E}+(D / V) R_{D}\)
WACC \(=.70(.1514)+.30(.08)\)
\(W A C C=.1300\) or \(13.00 \%\)
```

And the WACC with 60 percent debt is:

```
\(W A C C=(E / V) R_{E}+(D / V) R_{D}\)
WACC \(=.40(.2050)+.60(.08)\)
\(W A C C=.1300\) or \(13.00 \%\)
```

12. Using M\&M Proposition I with taxes, the value of the levered firm is:
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}$
$\mathrm{V}_{\mathrm{L}}=\$ 540,000+.35(\$ 110,000)$
$\mathrm{V}_{\mathrm{L}}=\$ 578,500$
13. The interest tax shield is the total interest paid times the tax rate, so:

Interest tax shield $=$ Interest $\operatorname{paid}\left(\mathrm{t}_{\mathrm{C}}\right)$
Interest tax shield $=\$ 45,000,000(.38)$
Interest tax shield $=\$ 17,100,000$
The interest tax shield represents the tax savings in current income due to the deductibility of a firm's qualified debt expenses.

## Intermediate

14. M\&M Proposition I with no taxes states the value of the levered firm is equal to the value of the unlevered firm. So, with no taxes, the value of the firm is it issues the debt is:
$\mathrm{V}_{\mathrm{U}}=\mathrm{V}_{\mathrm{L}}=\$ 90,000,000$
With corporate taxes, we need to use M\&M Proposition I with taxes, so the value of the firm is:
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}$
$\mathrm{V}_{\mathrm{L}}=\$ 90,000,000+.40(\$ 30,000,000)$
$\mathrm{V}_{\mathrm{L}}=\$ 102,000,000$
15. The value of the firm is the value of the debt plus the value of the equity. We can use this relationship to find the value of equity in each case. So, the debt-equity ratio with no taxes is:
$\mathrm{V}=\mathrm{E}+\mathrm{D}$
$\$ 90,000,000=\mathrm{E}+30,000,000$
$\mathrm{E}=\$ 60,000,000$

So, the debt-equity ratio is:
Debt-equity ratio $=\$ 30,000,000 / \$ 60,000,000$
Debt-equity ratio $=.50$
With taxes, the debt-equity ratio becomes:
$\mathrm{V}=\mathrm{E}+\mathrm{D}$
$\$ 102,000,000=\mathrm{E}+30,000,000$
$\mathrm{E}=\$ 72,000,000$
So, the debt-equity ratio is:
Debt-equity ratio $=\$ 30,000,000 / \$ 72,000,000$
Debt-equity ratio $=.42$
16. When the company is all-equity financed, the cost of equity is:
$\mathrm{WACC}=\mathrm{R}_{\mathrm{E}}=.095$ or $9.50 \%$
Using M\&M Proposition I with no taxes, the cost of equity will be:
$\mathrm{R}_{\mathrm{E}}=\mathrm{R}_{\mathrm{A}}+\left(\mathrm{R}_{\mathrm{A}}-\mathrm{R}_{\mathrm{D}}\right)(\mathrm{D} / \mathrm{E})$
$\mathrm{R}_{\mathrm{E}}=.095+(.095-.072)(1)$
$\mathrm{R}_{\mathrm{E}}=.1180$ or $11.80 \%$
And the new WACC will be:
$W A C C=(E / V) R_{E}+(D / V) R_{D}$
WACC $=(.50) .1180+(.50)(.072)$
WACC $=.0950$ or $9.50 \%$

## Challenge

17. With no debt, we are finding the value of an unlevered firm, so:
$\mathrm{V}_{\mathrm{U}}=\operatorname{EBIT}\left(1-\mathrm{T}_{\mathrm{C}}\right) / \mathrm{R}_{\mathrm{U}}$
$\mathrm{V}_{\mathrm{U}}=\$ 35,000(1-.38) / .14$
$\mathrm{V}_{\mathrm{U}}=\$ 155,000.00$
With debt, we simply need to use the equation for the value of a levered firm. With 50 percent debt, 50 percent of the firm value after recapitalization will be debt. We can find the value of the levered firm using M\&M Proposition I with taxes, which will be:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D} \\
& \mathrm{~V}_{\mathrm{L}}=\$ 155,000+.38(.50) \mathrm{V}_{\mathrm{L}} \\
& .81 \mathrm{~V}_{\mathrm{L}}=\$ 155,000 \\
& \mathrm{~V}_{\mathrm{L}}=\$ 191,358.02
\end{aligned}
$$

With 75 percent debt, the value of the firm is:

```
\(\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}\)
\(\mathrm{V}_{\mathrm{L}}=\$ 155,000+.38(.75) \mathrm{V}_{\mathrm{L}}\)
\(.715 \mathrm{~V}_{\mathrm{L}}=\$ 155,000\)
\(\mathrm{V}_{\mathrm{L}}=\$ 216,783.22\)
```

And with 100 percent debt, the value of the firm is:

```
\(\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}\)
\(\mathrm{V}_{\mathrm{L}}=\$ 155,000+.38\left(\mathrm{~V}_{\mathrm{L}}\right)\)
\(.62 \mathrm{~V}_{\mathrm{L}}=\$ 155,000\)
\(\mathrm{V}_{\mathrm{L}}=\$ 250,000\)
```

As the debt-equity ratio increases, $M \& M$ Proposition I shows that the value of the firm increases.
18. The return on equity is net income divided by equity. Net income can be expressed as:
$\mathrm{NI}=\left(\mathrm{EBIT}-\mathrm{R}_{\mathrm{D}} \mathrm{D}\right)\left(1-\mathrm{T}_{\mathrm{C}}\right)$

So, ROE is:
$R_{E}=\left(E B I T-R_{D} D\right)\left(1-T_{C}\right) / E$

Now we can rearrange and substitute as follows to arrive at M\&M Proposition II with taxes:
$\mathrm{R}_{\mathrm{E}}=\left[\mathrm{EBIT}\left(1-\mathrm{T}_{\mathrm{C}}\right) / \mathrm{E}\right]-\left[\mathrm{R}_{\mathrm{D}}(\mathrm{D} / \mathrm{E})\left(1-\mathrm{T}_{\mathrm{C}}\right)\right]$
$\mathrm{R}_{\mathrm{E}}=\mathrm{R}_{\mathrm{A}} \mathrm{V}_{\mathrm{U}} / \mathrm{E}-\left[\mathrm{R}_{\mathrm{D}}(\mathrm{D} / \mathrm{E})\left(1-\mathrm{T}_{\mathrm{C}}\right)\right]$
$R_{E}=R_{A}\left(V_{L}-T_{C} D\right) / E-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
$R_{E}=R_{A}\left(E+D-T_{C} D\right) / E-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
$R_{E}=R_{A}+\left(R_{A}-R_{D}\right)(D / E)\left(1-T_{C}\right)$

And the equation for WACC is:
$\mathrm{WACC}=(\mathrm{E} / \mathrm{V}) \mathrm{R}_{\mathrm{E}}+(\mathrm{D} / \mathrm{V}) \mathrm{R}_{\mathrm{D}}\left(1-\mathrm{T}_{\mathrm{C}}\right)$
Substituting the $\mathrm{M} \& \mathrm{M}$ Proposition II equation into the equation for WACC, we get:
$\mathrm{WACC}=(\mathrm{E} / \mathrm{V})\left[\mathrm{R}_{\mathrm{A}}+\left(\mathrm{R}_{\mathrm{A}}-\mathrm{R}_{\mathrm{D}}\right)(\mathrm{D} / \mathrm{E})\left(1-\mathrm{T}_{\mathrm{C}}\right)\right]+(\mathrm{D} / \mathrm{V}) \mathrm{R}_{\mathrm{D}}\left(1-\mathrm{T}_{\mathrm{C}}\right)$

Rearranging and reducing the equation, we get:

```
\(W A C C=R_{A}\left[(E / V)+(E / V)(D / E)\left(1-T_{C}\right)\right]+R_{D}\left(1-T_{C}\right)[(D / V)-(E / V)(D / E)]\)
\(W A C C=R_{A}\left[(E / V)+(D / V)\left(1-T_{C}\right)\right]\)
\(W A C C=R_{A}\left[\{(E+D) / V\}-T_{C}(D / V)\right]\)
\(\mathrm{WACC}=\mathrm{R}_{\mathrm{A}}\left[1-\mathrm{T}_{\mathrm{C}}(\mathrm{D} / \mathrm{V})\right]\)
```

Thus, with a debt-to-value ratio of 1 , the WACC is $\mathrm{R}_{\mathrm{A}}\left[1-\mathrm{T}_{\mathrm{C}}\right]$. This result makes intuitive sense. It says that the cost of capital is the same as it would be for an all equity firm, but with the tax benefit of debt.

To find the value of the firm, M\&M Proposition I with taxes states:
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}$
Since the firm is entirely financed by debt, the value of the firm must be equal to the amount of debt, so:
$\mathrm{V}_{\mathrm{L}}=\mathrm{D}$
Substituting, we get:
$\mathrm{D}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \mathrm{D}$
$\mathrm{D}-\mathrm{T}_{\mathrm{C}} \mathrm{D}=\mathrm{V}_{\mathrm{U}}$
$\mathrm{D}\left(1-\mathrm{T}_{\mathrm{C}}\right)=\mathrm{V}_{\mathrm{U}}$
$\mathrm{D}=\mathrm{V}_{\mathrm{U}} /\left(1-\mathrm{T}_{\mathrm{C}}\right)$
$\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}} /\left(1-\mathrm{T}_{\mathrm{C}}\right)$
Again, this result makes intuitive sense. The value of the firm is equal to its all-equity value grossed up by the tax benefit of debt.
19. The return on equity is net income divided by equity. Net income can be expressed as:
$\mathrm{NI}=\left(\mathrm{EBIT}-\mathrm{R}_{\mathrm{D}} \mathrm{D}\right)\left(1-\mathrm{T}_{\mathrm{C}}\right)$
So, ROE is:
$R_{E}=\left(E B I T-R_{D} D\right)\left(1-T_{C}\right) / E$
Now we can rearrange and substitute as follows:
$R_{E}=\left[E B I T\left(1-T_{C}\right) / E\right]-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
If we assume that EBIT and $\mathrm{T}_{\mathrm{C}}$ are constant, then we can treat the unlevered firm as a perpetuity whose per-period cashflows are $\operatorname{EBIT}\left(1-T_{C}\right)$. Then:.
$\mathrm{V}_{\mathrm{U}}=\operatorname{EBIT}\left(1-\mathrm{T}_{\mathrm{C}}\right) / \mathrm{R}_{\mathrm{A}}$
Substituting, we get:.
$R_{E}=R_{A} V_{U} / E-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
Note that we've implicitly assumed, with $M \& M$, that $R_{A}$ is independent of capital structure, so that $\mathrm{R}_{\mathrm{A}}$ of the unlevered firm can be treated as the required return on assets of the leveraged firm.

## B-252 SOLUTIONS

Recalling, from $M \& M$ Proposition $I$ with taxes that $V_{L}=V_{U}+T_{C} D$, rearranging, and substituting, we get:
$R_{E}=R_{A}\left(V_{L}-T_{C} D\right) / E-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
$R_{E}=R_{A}\left(E+D-t_{C} D\right) / E-\left[R_{D}(D / E)\left(1-T_{C}\right)\right]$
$R_{E}=R_{A}+\left(R_{A}-R_{D}\right)(D / E)\left(1-T_{C}\right)$
This result is known as M\&M Proposition II with taxes.
20. M\&M Proposition II, with no taxes is:
$R_{E}=R_{A}+\left(R_{A}-R_{f}\right)(D / E)$
Note that we assumed the return on debt was the risk-free rate. This is an important assumption of M\&M Proposition II. The CAPM to calculate the cost of equity is expressed as:
$R_{E}=\beta_{E}\left(R_{M}-R_{f}\right)+R_{f}$
We can rewrite the CAPM to express the return on an unlevered company as:
$R_{A}=\beta_{A}\left(R_{M}-R_{f}\right)+R_{f}$
We can now substitute the CAPM for an unlevered company into M\&M Proposition II. Doing so and rearranging the terms we get:
$R_{E}=\beta_{A}\left(R_{M}-R_{f}\right)+R_{f}+\left[\beta_{A}\left(R_{M}-R_{f}\right)+R_{f}-R_{f}\right](D / E)$
$R_{E}=\beta_{A}\left(R_{M}-R_{f}\right)+R_{f}+\left[\beta_{A}\left(R_{M}-R_{f}\right)\right](D / E)$
$\mathrm{R}_{\mathrm{E}}=(1+\mathrm{D} / \mathrm{E}) \beta_{\mathrm{A}}\left(\mathrm{R}_{\mathrm{M}}-\mathrm{R}_{\mathrm{f}}\right)+\mathrm{R}_{\mathrm{f}}$
Now we set this equation equal to the CAPM equation to calculate the cost of equity and reduce:
$\beta_{E}\left(R_{M}-R_{f}\right)+R_{f}=(1+D / E) \beta_{A}\left(R_{M}-R_{f}\right)+R_{f}$
$\beta_{E}\left(R_{M}-R_{f}\right)=(1+D / E) \beta_{A}\left(R_{M}-R_{f}\right)$
$\beta_{\mathrm{E}}=\beta_{\mathrm{A}}(1+\mathrm{D} / \mathrm{E})$

## CHAPTER 14 DIVIDENDS AND DIVIDEND POLICY

## Answers to Concepts Review and Critical Thinking Questions

1. Dividend policy deals with the timing of dividend payments, not the amounts ultimately paid. Dividend policy is irrelevant when the timing of dividend payments doesn't affect the present value of all future dividends.
2. A stock repurchase reduces equity while leaving debt unchanged. The debt ratio rises. A firm could, if desired, use excess cash to reduce debt instead. This is a capital structure decision.
3. The chief drawback to a strict dividend policy is the variability in dividend payments. This is a problem because investors tend to want a somewhat predictable cash flow. Also, if there is information content to dividend announcements, then the firm may be inadvertently telling the market that it is expecting a downturn in earnings prospects when it cuts a dividend, when in reality its prospects are very good. In a compromise policy, the firm maintains a relatively constant dividend. It increases dividends only when it expects earnings to remain at a sufficiently high level to pay the larger dividends, and it lowers the dividend only if it absolutely has to.
4. Friday, December 29 is the ex-dividend day. Remember not to count January 1 because it is a holiday, and the exchanges are closed. Anyone who buys the stock before December 29 is entitled to the dividend, assuming they do not sell it again before December 29.
5. No, because the money could be better invested in stocks that pay dividends in cash that will benefit the fundholders directly.
6. The change in price is due to the change in dividends, not to the change in dividend policy. Dividend policy can still be irrelevant without a contradiction.
7. The stock price dropped because of an expected drop in future dividends. Since the stock price is the present value of all future dividend payments, if the expected future dividend payments decrease, then the stock price will decline.
8. The plan will probably have little effect on shareholder wealth. The shareholders can reinvest on their own, and the shareholders must pay the taxes on the dividends either way. However, the shareholders who take the option may benefit at the expense of the ones who don't (because of the discount). Also as a result of the plan, the firm will be able to raise equity by paying a $10 \%$ flotation cost (the discount), which may be a smaller discount than the market flotation costs of a new issue.
9. If these firms just went public, they probably did so because they were growing and needed the additional capital. Growth firms typically pay very small cash dividends, if they pay a dividend at all. This is because they have numerous projects, and therefore reinvest the earnings in the firm instead of paying cash dividends.

## B-254 SOLUTIONS

10. It would not be irrational to find low-dividend, high-growth stocks. The trust should be indifferent between receiving dividends or capital gains since it does not pay taxes on either one (ignoring possible restrictions on invasion of principal, etc.). It would be irrational, however, to hold municipal bonds. Since the trust does not pay taxes on the interest income it receives, it does not need the tax break associated with the municipal bonds. Therefore, it should prefer to hold higher yielding, taxable bonds.

## Solutions to Questions and Problems

## Basic

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. With no taxes we would expect the stock price to drop by exactly the amount of the dividend, so the new stock price will be:

New stock price $=\$ 96.00-2.50$
New stock price $=\$ 93.50$
Your total stock investment will be worth:
Stock value $=200 \times \$ 93.50$
Stock value $=\$ 18,700$
2. Your total portfolio value will be the total stock value plus the dividends received, so:

Portfolio value $=\$ 18,700+(200 \times \$ 2.50)$
Portfolio value $=\$ 19,200$
3. The aftertax dividend is the pretax dividend times one minus the tax rate, so:

Aftertax dividend $=\$ 6.40$ (1-.15)
Aftertax dividend $=\$ 5.44$
The stock price should drop by the aftertax dividend amount, or:
Ex-dividend price $=\$ 108.00-5.44$
Ex-dividend price $=\$ 102.56$
4. a. The shares outstanding increases by 10 percent, so:

New shares outstanding $=25,000(1.10)$
New shares outstanding $=27,500$
New shares issued $=2,500$

Since the par value of the new shares is $\$ 1$, the capital surplus per share is $\$ 34$. The total capital surplus is therefore:

Capital surplus on new shares $=2,500(\$ 34)$
Capital surplus on new shares $=\$ 85,000$
The new equity account balances will be:

| Common stock (\$1 par value) | $\$ 27,500$ |
| :--- | ---: |
| Capital surplus | 255,000 |
| Retained earnings | $\underline{442,500}$ |
|  | $\underline{\$ 725,000}$ |

b. The shares outstanding increases by 25 percent, so:

New shares outstanding $=25,000(1.25)$
New shares outstanding $=31,250$
New shares issued $=6,250$
Since the par value of the new shares is $\$ 1$, the capital surplus per share is $\$ 34$. The total capital surplus is therefore:

Capital surplus on new shares $=6,250(\$ 34)$
Capital surplus on new shares $=\$ 212,500$
The new equity account balances will be:

| Common stock (\$1 par value) | $\$ 31,250$ |
| :--- | ---: |
| Capital surplus | 382,500 |
| Retained earnings | $\underline{311,250}$ |
|  | $\underline{\$ 725,000}$ |

5. a. To find the new shares outstanding, we multiply the current shares outstanding times the ratio of new shares to old shares, so:

New shares outstanding $=25,000(2 / 1)$
New shares outstanding $=50,000$
The equity accounts are unchanged except that the par value of the stock is changed by the ratio of new shares to old shares, so the new par value is:

New par value $=\$ 1(1 / 2)$
New par value $=\$ 0.50$ per share

## B-256 SOLUTIONS

b. To find the new shares outstanding, we multiply the current shares outstanding times the ratio of new shares to old shares, so:

New shares outstanding $=25,000(1 / 5)$
New shares outstanding $=5,000$
The equity accounts are unchanged except the par value of the stock is changed by the ratio of new shares to old shares, so the new par value is:

New par value $=\$ 1(5 / 1)$
New par value $=\$ 5.00$ per share
6. To find the new stock price, we multiply the current stock price by the ratio of old shares to new shares, so:
a. $\quad \$ 92(3 / 5)=\$ 55.20$
b. $\quad \$ 92(1 / 1.15)=\$ 80.00$
c. $\quad \$ 92(1 / 1.425)=\$ 64.56$
d. $\quad \$ 92(7 / 4)=\$ 161.00$
e. To find the new shares outstanding, we multiply the current shares outstanding times the ratio of new shares to old shares, so:
a: $400,000(5 / 3)=666,667$
b: $400,000(1.15)=460,000$
c: $400,000(1.425)=570,000$
d: $400,000(4 / 7)=228,571$
7. The stock price is the total market value of equity divided by the shares outstanding, so:
$\mathrm{P}_{0}=\$ 415,000$ equity $/ 35,000$ shares
$\mathrm{P}_{0}=\$ 11.86$ per share
Ignoring tax effects, the stock price will drop by the amount of the dividend, so:
$\mathrm{P}_{\mathrm{X}}=\$ 11.86-1.10$
$\mathrm{P}_{\mathrm{X}}=\$ 10.76$
The total dividends paid will be:
$\$ 1.10$ per share $(35,000$ shares $)=\$ 38,500$

The equity and cash accounts will both decline by $\$ 35,000$. The new balance sheet will be:

|  | Market Value Balance Sheet |  |  |
| :--- | ---: | ---: | ---: |
| Cash | $\$ 51,500$ | Equity | $\$ 376,500$ |
| Fixed assets | 325,000 |  |  |
|  | $\$ 376,500$ Total | $\$ 376,500$ <br> Total |  |

8. Repurchasing the shares will reduce shareholders' equity by $\$ 38,500$. The shares repurchased will be the total purchase amount divided by the stock price, so:

Shares bought $=\$ 38,500 / \$ 11.86$
Shares bought $=3,247$
And the new shares outstanding will be:
New shares outstanding $=35,000-3,247$
New shares outstanding $=31,753$
After repurchase, the new stock price is:
Share price $=(\$ 415,000-38,500) / 31,753$ shares
Share price $=\$ 11.86$
The repurchase is effectively the same as the cash dividend because you either hold a share worth $\$ 11.86$, or a share worth $\$ 10.76$ and $\$ 1.10$ in cash. Therefore, if you participate in the repurchase according to the dividend payout percentage, you are unaffected.
9. The stock price is the total market value of equity divided by the shares outstanding, so:
$\mathrm{P}_{0}=\$ 555,000$ equity $/ 15,000$ shares
$\mathrm{P}_{0}=\$ 37$ per share
The shares outstanding will increase by 20 percent, so:
New shares outstanding $=15,000(1.20)$
New shares outstanding $=18,000$
The new stock price is the market value of equity divided by the new shares outstanding, so:
$\mathrm{P}_{\mathrm{X}}=\$ 555,000 / 18,000$ shares
$\mathrm{P}_{\mathrm{X}}=\$ 30.83$
10. With a stock dividend, the shares outstanding will increase by one plus the dividend percentage, so:

New shares outstanding $=425,000(1.10)$
New shares outstanding $=467,500$

## B-258 SOLUTIONS

The capital surplus is the capital paid in excess of par value, which is $\$ 1$, so:

Capital surplus for new shares $=42,500(\$ 47)$
Capital surplus for new shares $=\$ 1,977,500$
The new capital surplus will be the old capital surplus plus the additional capital surplus for the new shares, so:

Capital surplus $=\$ 1,430,000+1,977,500$
Capital surplus $=\$ 3,427,500$
The new equity portion of the balance sheet will look like this:

$$
\begin{array}{lr}
\text { Common stock (\$1 par value) } & \$ 467,500 \\
\text { Capital surplus } & 3,427,500 \\
\text { Retained earnings } & \underline{1,210,000} \\
& \underline{\$ 5,105,000} \\
\hline
\end{array}
$$

11. The only equity account that will be affected is the par value of the stock. The par value will change by the ratio of old shares to new shares, so:

New par value $=\$ 1(1 / 2)$
New par value $=\$ 0.50$ per share
The total dividends paid this year will be the dividend amount times the number of shares outstanding. The company had 425,000 shares outstanding before the split. We must remember to adjust the shares outstanding for the stock split, so:

Total dividends paid this year $=\$ 0.72(425,000$ shares $)(2 / 1 \mathrm{split})$
Total dividends paid this year $=\$ 612,000$
The dividends increased by 10 percent, so the total dividends paid last year were:
Last year's dividends $=\$ 612,000 / 1.10$
Last year's dividends $=\$ 556,363.64$

And to find the dividends per share, we simply divide this amount by the shares outstanding last year. Doing so, we get:

Dividends per share last year $=\$ 556,363.64 / 425,000$ shares
Dividends per share last year $=\$ 1.31$
12. The equity portion of capital outlays is the retained earnings. Subtracting dividends from net income, we get:

Equity portion of capital outlays $=\$ 3,800-425$
Equity portion of capital outlays $=\$ 3,375$
Since the debt-equity ratio is 1.20 , we can find the new borrowings for the company by multiplying the equity investment by the debt-equity ratio, so:

New borrowings $=1.20(\$ 3,375)$
Net borrowings $=\$ 4,050$
And the total capital outlay will be the sum of the new equity and the new debt, which is:
Total capital outlays $=\$ 4,050+3,375$
Total capital outlays $=\$ 7,425$
13. $a$. The payout ratio is the dividend per share divided by the earnings per share, so:

Payout ratio $=\$ 1.60 / \$ 7.62$
Payout ratio $=.2100$ or $21.00 \%$
b. Under a residual dividend policy, the additions to retained earnings, which is the equity portion of the planned capital outlays, is the retained earnings per share times the number of shares outstanding, so:

Equity portion of capital outlays $=6,500,000$ shares $(\$ 7.62-1.60)$
Equity portion of capital outlays $=\$ 39,130,000$
The debt-equity ratio is the new borrowing divided by the new equity, so:
D/E ratio $=\$ 19,000,000 / \$ 39,130,000$
$\mathrm{D} / \mathrm{E}$ ratio $=.4856$
14. a. Since the company has a debt-equity ratio of 1.4 , they can raise $\$ 1.40$ in debt for every $\$ 1$ of equity. The maximum capital outlay with no outside equity financing is:

Maximum capital outlay $=\$ 2,100,000+1.4(\$ 2,100,000)$
Maximum capital outlay $=\$ 5,040,000$
b. If planned capital spending is $\$ 6$ million, then no dividend will be paid and new equity will be issued since this exceeds the amount calculated in $a$.
c. No, they do not maintain a constant dividend payout because, with the strict residual policy, the dividend will depend on the investment opportunities and earnings. As these two things vary, the dividend payout will also vary.

## B-260 SOLUTIONS

15. a. We can find the new borrowings for the company by multiplying the equity investment by the debt-equity ratio, so we get:

New debt $=.9(\$ 43,000,000)$
New debt $=\$ 38,700,000$
Adding the new retained earnings, we get:
Maximum investment with no outside equity financing $=\$ 43,000,000+38,700,000$
Maximum investment with no outside equity financing $=\$ 81,700,000$
b. A debt-equity ratio of .9 implies capital structure is $.9 / 1.9$ debt and $1 / 1.9$ equity. The equity portion of the planned new investment will be:

Equity portion of investment funds $=1 / 1.9(\$ 50,000,000)$
Equity portion of investment funds $=\$ 26,315,789$
This is the addition to retained earnings, so the total available for dividend payments is:
Residual $=\$ 43,000,000-26,315,789$
Residual $=\$ 16,684,211$
This makes the dividend per share:
Dividend per share $=\$ 16,684,211 / 8,000,000$ shares
Dividend per share $=\$ 2.09$
c. The borrowing will be:

Borrowing $=\$ 50,000,000-26,315,789$
Borrowing = \$23,684,211
Alternatively, we could calculate the new borrowing as the weight of debt in the capital structure times the planned capital outlays, so:

Borrowing $=.9 / 1.9(\$ 50,000,000)$
Borrowing $=\$ 23,684,211$
The addition to retained earnings is $\$ 26,315,789$, which we calculated in part $b$.
d. If the company plans no capital outlays, no new borrowing will take place. The dividend per share will be:

Dividend per share $=\$ 43,000,000 / 8,000,000$ shares
Dividend per share $=\$ 5.38$

## Intermediate

16. The price of the stock today is the PV of the dividends, so:
$\mathrm{P}_{0}=\$ 3.50 / 1.15+\$ 45 / 1.15^{2}$
$\mathrm{P}_{0}=\$ 37.07$
To find the equal two year dividends with the same present value as the price of the stock, we set up the following equation and solve for the dividend (Note: The dividend is a two-year annuity, so we could solve with the annuity factor as well):
$\$ 37.07=\mathrm{D} / 1.15+\mathrm{D} / 1.15^{2}$
$\mathrm{D}=\$ 22.80$
We now know the cash flow per share we want each of the next two years. We can find the price of stock in one year, which will be:
$\mathrm{P}_{1}=\$ 45 / 1.15=\$ 39.13$
Since you own 1,000 shares, in one year you want:
Cash flow in Year $1=1,000(\$ 22.80)$
Cash flow in Year $1=\$ 22,802.33$
But you'll only get:
Dividends received in one year $=1,000(\$ 3.50)$
Dividends received in one year $=\$ 3,500.00$
Thus, in one year you will need to sell additional shares in order to increase your cash flow. The number of shares to sell in year one is:

Shares to sell at time one $=(\$ 22,802.33-3,500) / \$ 39.13$
Shares to sell at time one $=493.28$ shares
At Year 2, you cash flow will be the dividend payment times the number of shares you still own, so the Year 2 cash flow is:

Year 2 cash flow $=\$ 45(1,000-493.28)$
Year 2 cash flow $=\$ 22,802.33$

## B-262 SOLUTIONS

17. If you only want $\$ 2,000$ in Year 1 , you will buy:
$(\$ 3,500-2,000) / \$ 39.13=38.33$ shares
at time 1. Your dividend payment in Year 2 will be:
Year 2 dividend $=(1,000+38.33)(\$ 45)$
Year 2 dividend $=\$ 46,725$
Note, the present value of each cash flow stream is the same. Below, we show this by finding the present values as:
$\mathrm{PV}=\$ 2,000 / 1.15+\$ 46,725 / 1.15^{2}$
$\mathrm{PV}=\$ 37,069.94$
$\mathrm{PV}=1,000(\$ 3.50) / 1.15+1,000(\$ 45) / 1.15^{2}$
$\mathrm{PV}=\$ 37,069.94$
18. a. If the company makes a dividend payment, we can calculate the wealth of a shareholder as:

Dividend per share $=\$ 15,000 / 2,800$ shares
Dividend per share $=\$ 5.36$
The stock price after the dividend payment will be:
$\mathrm{P}_{\mathrm{X}}=\$ 70-5.36$
$\mathrm{P}_{\mathrm{X}}=\$ 64.64$ per share
The shareholder will have a stock worth $\$ 64.64$ and a $\$ 5.36$ dividend for a total wealth of $\$ 70$. If the company makes a repurchase, the company will repurchase:

Shares repurchased $=\$ 15,000 / \$ 70$
Shares repurchased $=214.29$ shares
If the shareholder lets their shares be repurchased, they will have $\$ 70$ in cash. If the shareholder keeps their shares, they're still worth $\$ 70$.
b. If the company pays dividends, the current EPS is $\$ 2.60$, and the P/E ratio is:
$\mathrm{P} / \mathrm{E}=\$ 64.64 / \$ 2.60$
$\mathrm{P} / \mathrm{E}=24.86$
If the company repurchases stock, the number of shares will decrease. The total net income is the EPS times the current number of shares outstanding. Dividing net income by the new number of shares outstanding, we find the EPS under the repurchase is:

EPS $=\$ 2.60(2,800) /(2,800-216.67)$
EPS $=\$ 2.82$

The stock price will remain at $\$ 70$ per share, so the $\mathrm{P} / \mathrm{E}$ ratio is:

$$
\begin{aligned}
& \mathrm{P} / \mathrm{E}=\$ 70 / \$ 2.82 \\
& \mathrm{P} / \mathrm{E}=24.86
\end{aligned}
$$

c. A share repurchase would seem to be the preferred course of action. Only those shareholders who wish to sell will do so, giving the shareholder a tax timing option that he or she doesn't get with a dividend payment.

## Challenge

19. Assuming no capital gains tax, the aftertax return for the Gordon Company is the capital gains growth rate, plus the dividend yield times one minus the tax rate. Using the constant growth dividend model, we get:

Aftertax return $=\mathrm{g}+\mathrm{D}(1-\mathrm{t})=.15$
Solving for g , we get:
$.15=\mathrm{g}+.05(1-.35)$
$\mathrm{g}=.1175$
The equivalent pretax return for Gecko Company, which pays no dividend, is:
Pretax return $=\mathrm{g}+\mathrm{D}=.1175+.05=.1675$ or $16.75 \%$
20. Using the equation for the decline in the stock price ex-dividend for each of the tax rate policies, we get:
$\left(\mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}\right) / \mathrm{D}=\left(1-\mathrm{T}_{\mathrm{P}}\right) /\left(1-\mathrm{T}_{\mathrm{G}}\right)$
a. $\quad \mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=\mathrm{D}(1-0) /(1-0)$
$\mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=\mathrm{D}$
b. $\quad \mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=\mathrm{D}(1-.15) /(1-0)$
$\mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=.85 \mathrm{D}$
c. $\quad \mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=\mathrm{D}(1-.15) /(1-.35)$
$\mathrm{P}_{0}-\mathrm{P}_{\mathrm{x}}=1.2143 \mathrm{D}$
d. With this tax policy, we simply need to multiply the personal tax rate times one minus the dividend exemption percentage, so:
$\mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=\mathrm{D}[1-(.35)(.30)] /(1-.65)$
$\mathrm{P}_{0}-\mathrm{P}_{\mathrm{X}}=1.377 \mathrm{D}$
e. Since different investors have widely varying tax rates on ordinary income and capital gains, dividend payments have different after-tax implications for different investors. This differential taxation among investors is one aspect of what we have called the clientele effect.

## CHAPTER 15 <br> RASING CAPITAL

## Answers to Concepts Review and Critical Thinking Questions

1. A company's internally generated cash flow provides a source of equity financing. For a profitable company, outside equity may never be needed. Debt issues are larger because large companies have the greatest access to public debt markets (small companies tend to borrow more from private lenders). Equity issuers are frequently small companies going public; such issues are often quite small.
2. From the previous question, economies of scale are part of the answer. Beyond this, debt issues are simply easier and less risky to sell from an investment bank's perspective. The two main reasons are that very large amounts of debt securities can be sold to a relatively small number of buyers, particularly large institutional buyers, such as pension funds and insurance companies, and debt securities are much easier to price.
3. They are riskier and harder to market from an investment bank's perspective.
4. Yields on comparable bonds can usually be readily observed, so pricing a bond issue accurately is much less difficult.
5. It is clear that the stock was sold too cheaply, so Eyetech had reason to be unhappy.
6. No, but, in fairness, pricing the stock in such a situation is extremely difficult.
7. It's an important factor. Only 6.5 million of the shares were underpriced. The other 32 million were, in effect, priced completely correctly.
8. He could have done worse since his access to the oversubscribed and, presumably, underpriced issues was restricted, while the bulk of his funds were allocated in stocks from the undersubscribed and, quite possibly, overpriced issues.
9. $a$. The price will probably go up because IPOs are generally underpriced. This is especially true for smaller issues, such as this one.
b. It is probably safe to assume that they are having trouble moving the issue, and it is likely that the issue is not substantially underpriced.

## Solutions to Questions and Problems

## Basic

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. If you receive 1,000 shares of each, the profit is:

Profit $=1,000(\$ 7)-1,000(\$ 3.50)$
Profit $=\$ 3,500$
Since you will only receive one-half of the shares of the oversubscribed issue, your profit will be:
Expected profit $=500(\$ 7)-1,000(\$ 3.50)$
Expected profit $=\$ 0$
This is an example of the winner's curse.
2. Using $X$ to stand for the required sale proceeds, the equation to calculate the total sale proceeds, including floatation costs is:
$\mathrm{X}(1-.08)=\$ 80,000,000$
$X=\$ 86,956,522$ required total proceeds from sale.
So the number of shares offered is the total amount raised divided by the offer price, which is:
Number of shares offered $=\$ 86,956,522 / \$ 45$
Number of shares offered $=1,932,367$
3. This is basically the same as the previous problem, except that we need to include the $\$ 450,000$ of expenses in the amount the company needs to raise, so:
$\mathrm{X}(1-.08)=\$ 80,450,000$
$X=\$ 87,445,652$ required total proceeds from sale.
Number of shares offered $=\$ 87,445,652 / \$ 45$
Number of shares offered $=1,943,237$
4. We need to calculate the net amount raised and the costs associated with the offer. The net amount raised is the number of shares offered times the price received by the company, minus the costs associated with the offer, so:

Net amount raised $=(4,100,000$ shares $)(\$ 22.00)-780,000-250,000$
Net amount raised $=\$ 89,170,000$

## B-266 SOLUTIONS

The company received $\$ 89,170,000$ from the stock offering. Now, we can calculate the direct costs. Part of the direct costs are given in the problem, but the company also had to pay the underwriters. The stock was offered at $\$ 23.65$ per share, and the company received $\$ 22.00$ per share. The difference, which is the underwriters spread, is also a direct cost. The total direct costs were:

Total direct costs $=\$ 780,000+(\$ 23.65-22.00)(4,100,000$ shares $)$
Total direct costs $=\$ 7,545,000$
We are given part of the indirect costs in the problem. Another indirect cost is the immediate price appreciation. The total indirect costs were:

Total indirect costs $=\$ 250,000+(\$ 28.00-23.65)(4,100,000$ shares $)$
Total indirect costs $=\$ 18,085,000$
This makes the total costs:
Total costs $=\$ 7,545,000+18,085,000$
Total costs $=\$ 25,630,000$
The floatation costs as a percentage of the amount raised is the total cost divided by the amount raised, so:

Floatation cost percentage $=\$ 25,630,000 / \$ 89,170,000$
Floatation cost percentage $=.2874$ or $28.74 \%$
5. Using $X$ to stand for the required sale proceeds, the equation to calculate the total sale proceeds, including floatation costs is:
$\mathrm{X}(1-.06)=\$ 45,000,000$
$X=\$ 47,872,340$ required total proceeds from sale.
So the number of shares offered is the total amount raised divided by the offer price, which is:
Number of shares offered $=\$ 47,872,340 / \$ 32$
Number of shares offered $=1,496,011$
6. This is basically the same as the previous problem, except that we need to include the $\$ 1,450,000$ of expenses in the amount the company needs to raise, so:
$\mathrm{X}(1-.06)=\$ 46,450,000$
$\mathrm{X}=\$ 49,414,894$ required total proceeds from sale.
Number of shares offered $=\$ 49,414,894 / \$ 32$
Number of shares offered $=1,544,215$
7. We need to calculate the net amount raised and the costs associated with the offer. The net amount raised is the number of shares offered times the price received by the company, minus the costs associated with the offer, so:

Net amount raised $=(6,500,000$ shares $)(\$ 19.00)-900,000-175,000$
Net amount raised $=\$ 122,425,000$

The company received $\$ 122,425,000$ from the stock offering. Now, we can calculate the direct costs. Part of the direct costs are given in the problem, but the company also had to pay the underwriters. The stock was offered at $\$ 21.25$ per share, and the company received $\$ 19.00$ per share. The difference, which is the underwriters spread, is also a direct cost. The total direct costs were:

Total direct costs $=\$ 900,000+(\$ 21.25-19.00)(6,500,000$ shares $)$
Total direct costs $=\$ 15,525,000$
We are given part of the indirect costs in the problem. Another indirect cost is the immediate price appreciation. The total indirect costs were:

Total indirect costs $=\$ 175,000+(\$ 26.45-21.25)(6,500,000$ shares $)$
Total indirect costs $=\$ 33,975,000$

This makes the total costs:

Total costs $=\$ 15,525,000+33,975,000$
Total costs $=\$ 49,500,000$
The floatation costs as a percentage of the amount raised is the total cost divided by the amount raised, so:

Floatation cost percentage $=\$ 49,500,000 / \$ 122,425,000$
Floatation cost percentage $=.4043$ or $40.43 \%$

## CHAPTER 16 SHORT-TERM FINANCIAL PLANNING

## Answers to Concepts Review and Critical Thinking Questions

1. These are firms with relatively long inventory periods and/or relatively long receivables periods. Thus, such firms tend to keep inventory on hand, and they allow customers to purchase on credit and take a relatively long time to pay.
2. These are firms that have a relatively long time between the time purchased inventory is paid for and the time that inventory is sold and payment received. Thus, these are firms that have relatively short payables periods and/or relatively long receivable cycles.
3. a. Use: The cash balance declined by $\$ 200$ to pay the dividend.
b. Source: The cash balance increased by $\$ 500$ assuming the goods bought on payables credit were sold for cash.
c. Use: The cash balance declined by $\$ 900$ to pay for the fixed assets.
d. Use: The cash balance declined by $\$ 625$ to pay for the higher level of inventory.
$e$. Use: The cash balance declined by $\$ 1,200$ to pay for the redemption of debt.
4. Carrying costs will decrease because they are not holding goods in inventory. Shortage costs will probably increase, depending on how close the suppliers are and how well they can estimate need. The operating cycle will decrease because the inventory period is decreased.
5. Since the cash cycle equals the operating cycle minus the accounts payable period, it is not possible for the cash cycle to be longer than the operating cycle if the accounts payable is positive. Moreover, it is unlikely that the accounts payable period would ever be negative since that implies the firm pays its bills before they are incurred.
6. It lengthened its payables period, thereby shortening its cash cycle.
7. Their receivables period increased, thereby increasing their operating and cash cycles.
8. It is sometimes argued that large firms "take advantage of" smaller firms by threatening to take their business elsewhere. However, considering a move to another supplier to get better terms is the nature of competitive free enterprise.
9. They would like to! The payables period is a subject of much negotiation, and it is one aspect of the price a firm pays its suppliers. A firm will generally negotiate the best possible combination of payables period and price. Typically, suppliers provide strong financial incentives for rapid payment. This issue is discussed in detail in a later chapter on credit policy.
10. The company will need less financing because it is essentially borrowing more from its suppliers. Among other things, the company will likely need less short-term borrowing from other sources, so it will save on interest expense.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. a. No change. A dividend paid for by the sale of debt will not change cash since the cash raised from the debt offer goes immediately to shareholders.
b. No change. The real estate is paid for by the cash raised from the debt, so this will not change the cash balance.
c. No change. Inventory and accounts payable will increase, but neither will impact the cash account.
d. Decrease. The short-term bank loan is repaid with cash, which will reduce the cash balance.
e. Decrease. The payment of taxes is a cash transaction.
f. Decrease. The preferred stock will be repurchased with cash.
g. No change. Accounts receivable will increase, but cash will not increase until the sales are paid off.
h. Decrease. The interest is paid with cash, which will reduce the cash balance.
i. Increase. When payments for previous sales, or accounts receivable, are paid off, the cash balance increases since the payment must be made in cash.
j. Decrease. The accounts payable are reduced through cash payments to suppliers.
k. Decrease. Here, the dividend payments are made with cash, which is generally the case. This is different from part $a$, where debt was raised to make the dividend payment.
l. No change. The supplies are paid by the cash raised by the short-term note, so the cash balance will not change.
$m$. Decrease. The utility bills must be paid in cash.
n. Decrease. A cash payment will reduce cash.
o. Decrease. If marketable securities purchased, the company pay cash to the seller.

## B-270 SOLUTIONS

2. The total liabilities and equity of the company are the net book worth, or market value of equity, plus the long-term debt, so:

Total liabilities and equity $=\$ 41,000+6,800$
Total liabilities and equity $=\$ 47,800$
This is also equal to the total assets of the company. Since total assets are the sum of all assets, and cash is an asset, the cash account must be equal to total assets minus all other assets, so:

$$
\begin{aligned}
& \text { Cash }=\$ 47,800-38,500-4,100 \\
& \text { Cash }=\$ 5,200
\end{aligned}
$$

We have NWC other than cash, so the total NWC is:

$$
\begin{aligned}
& \text { NWC }=\$ 5,200+4,100 \\
& \mathrm{NWC}=\$ 9,300
\end{aligned}
$$

We can find total current assets by using the NWC equation. NWC is equal to:

$$
\begin{aligned}
& \mathrm{NWC}=\mathrm{CA}-\mathrm{CL} \\
& \$ 9,300=\mathrm{CA}-\$ 5,300 \\
& \mathrm{CA}=\$ 14,600
\end{aligned}
$$

3. a. Increase. If receivables go up, the time to collect the receivables would increase, which increases the operating cycle.
b. Increase. If credit repayment times are increased, customers will take longer to pay their bills, which will lead to an increase in the operating cycle.
c. Decrease. If the inventory turnover increases, the inventory period decreases.
d. No change. The accounts payable period is part of the cash cycle, not the operating cycle.
e. Decrease. If the receivables turnover increases, the receivables period decreases.
f. No change. Payments to suppliers affect the accounts payable period, which is part of the cash cycle, not the operating cycle.
4. a. Increase; Increase. If the terms of the cash discount are made less favorable to customers, the accounts receivable period will lengthen. This will increase both the cash cycle and the operating cycle.
b. Increase; No change. This will shorten the accounts payable period, which will increase the cash cycle. It will have no effect on the operating cycle since the length of the operating cycle is not affected by the payables period.
c. Decrease; Decrease. If more customers pay in cash, the accounts receivable period will decrease. This will decrease both the cash cycle and the operating cycle.
d. Decrease; Decrease. Fewer raw materials purchased will reduce the inventory period, which will decrease both the cash cycle and the operating cycle.
e. Decrease; No change. If more raw materials are purchased on credit, the accounts payable period will tend to increase, which would decrease the cash cycle. We should say that this may not be the case. The accounts payable period is a decision made by the company's management. The company could increase the accounts payable account and still make the payments in the same number of days. This would leave the accounts payable period unchanged, which would leave the cash cycle unchanged. The change in credit purchases made on credit will not affect the inventory period or the accounts receivable period, so the operating cycle will not change.
$f$. Increase; Increase. If more goods are produced for inventory, the inventory period will increase. This will increase both the cash cycle and operating cycle.
5. a. 45-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and ( $90-45$ )/90 $=1 / 2$ of current sales are collected.

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 340$ | $\$ 280$ | $\$ 315$ | $\$ 375$ |
| Sales | 560 | 630 | 750 | 905 |
| Cash collections | $\underline{-620}$ | $\underline{-595}$ | $\underline{\underline{-690}}$ | $\underline{\underline{-828}}$ |
| Ending receivables | $\underline{\underline{\$ 280}}$ | $\underline{\underline{\$ 315}}$ | $\underline{\underline{\$ 375}}$ | $\underline{\underline{\$ 453}}$ |

b. 60-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and $(90-60) / 90=1 / 3$ of current sales are collected.

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 340$ | $\$ 373$ | $\$ 420$ | $\$ 500$ |
| Sales | 560 | 630 | 750 | 905 |
| Cash collections | $\underline{-527}$ | $\underline{-583}$ | $\underline{-670}$ | $\underline{-802}$ |
| Ending receivables | $\underline{\underline{\$ 373}}$ | $\underline{\underline{\$ 420}}$ | $\underline{\underline{\$ 500}}$ | $\underline{\underline{\$ 603}}$ |

c. 30-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and $(90-30) / 90=2 / 3$ of current sales are collected.

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 340$ | $\$ 187$ | $\$ 210$ | $\$ 250$ |
| Sales | 560 | 630 | 750 | 905 |
| Cash collections | $\underline{-713}$ | $\underline{-607}$ | $\underline{-710}$ | $\underline{-853}$ |
| Ending receivables | $\underline{\$ 187}$ | $\underline{\$ 210}$ | $\underline{\$ 250}$ | $\underline{\underline{\$ 302}}$ |

## B-272 SOLUTIONS

6. The operating cycle is the inventory period plus the receivables period. The inventory turnover and inventory period are:

Inventory turnover $=$ COGS/Average inventory
Inventory turnover $=\$ 47,503 /\{[\$ 7,305+7,832] / 2\}$
Inventory turnover $=6.2764$ times
Inventory period $=365$ days/Inventory turnover
Inventory period $=365$ days/6.2764
Inventory period $=58.15$ days
And the receivables turnover and receivables period are:
Receivables turnover $=$ Credit sales/Average receivables
Receivables turnover $=\$ 96,125 /\{[\$ 3,241+3,621] / 2\}$
Receivables turnover $=28.0166$ times
Receivables period $=365$ days/Receivables turnover
Receivables period $=365$ days $/ 28.0166$
Receivables period $=13.03$ days
So, the operating cycle is:
Operating cycle $=58.15$ days +13.03 days
Operating cycle $=71.18$ days
The cash cycle is the operating cycle minus the payables period. The payables turnover and payables period are:

Payables turnover $=$ COGS/Average payables
Payables turnover $=\$ 47,503 /\{[\$ 4,795+5,105] / 2\}$
Payables turnover $=9.5966$ times
Payables period $=365$ days/Payables turnover
Payables period $=365$ days/9.5966
Payables period $=38.03$ days
So, the cash cycle is:
Cash cycle $=71.18$ days -38.03 days
Cash cycle $=33.15$ days
The firm is receiving cash on average 36.64 days after it pays its bills.
7. If we factor immediately, we receive cash on an average of 34 days sooner. The number of periods in a year are:

Number of periods $=365 / 53$
Number of periods $=6.8868$

The EAR of this arrangement is:

```
EAR \(=(1+\text { Periodic rate })^{\mathrm{m}}-1\)
EAR \(=(1+2.5 / 97.5)^{6.8868}-1\)
\(\mathrm{EAR}=.1905\) or \(19.05 \%\)
```

8. a. The payables period is zero since the company pays immediately. The payment in each period is 30 percent of next period's sales, so:

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :---: | :---: | :---: | :---: | :---: |
| Payment of accounts | $\$ 243.00$ | $\$ 252.00$ | $\$ 273.00$ | $\$ 269.10$ |

b. Since the payables period is 90 days, the payment in each period is 30 percent of the current period sales, so:

|  | Q1 | Q2 | Q3 | Q4 |
| :---: | :---: | :---: | :---: | :---: |
| Payment of accounts | $\$ 234.00$ | $\$ 243.00$ | $\$ 252.00$ | $\$ 273.00$ |

c. Since the payables period is 60 days, the payment in each period is $2 / 3$ of last quarter's orders, plus $1 / 3$ of this quarter's orders, or:

Quarterly payments $=2 / 3(.30)$ times current sales $+1 / 3(.30)$ next period sales.

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :---: | :---: | :---: | :---: | :---: |
| Payment of accounts | $\$ 237.00$ | $\$ 246.00$ | $\$ 259.00$ | $\$ 271.70$ |

9. Since the payables period is 60 days, payables in each period $=2 / 3$ of last quarter's orders, and $1 / 3$ of this quarter's orders, or $2 / 3(.75)$ times current sales $+1 / 3(.75)$ next period sales.

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Payment of accounts | $\$ 820.00$ | $\$ 907.50$ | $\$ 986.25$ | $\$ 983.75$ |
| Wages, taxes, other expenses | 315.00 | 354.00 | 381.00 | 421.50 |
| Long-term financing expenses | 40.00 | 40.00 | 40.00 | 40.00 |
| Total | $\$ 1,175.00$ | $\$ 1,301.50$ | $\$ 1,407.25$ | $\$ 1,445.25$ |

10. a. The November sales must have been the total uncollected sales minus the uncollected sales from December, divided by the collection rate two months after the sale, so:

November sales $=(\$ 90,000-64,000) / 0.15$
November sales $=\$ 173,333.33$

## B-274 SOLUTIONS

b. The December sales are the uncollected sales from December divided by the collection rate of the previous months' sales, so:

December sales $=\$ 64,000 / 0.35$
December sales $=\$ 182,857.14$
c. The collections each month for this company are:

```
Collections \(=.15(\) Sales from 2 months ago \()+.20(\) Last month's sales \()+.65\) (Current sales)
January collections \(=.15(\$ 173,333.33)+.20(\$ 182,857.14)+.65(\$ 147,000)\)
January collections \(=\$ 158,121.43\)
February collections \(=.15(\$ 182,857.14)+.20(\$ 147,000)+.65(\$ 169,000)\)
February collections = \$166,678.57
March collections \(=.15(\$ 147,000)+.20(\$ 169,000)+.65(\$ 187,000)\)
March collections \(=\$ 177,400.00\)
```

11. The sales collections each month will be:

Sales collections $=.35$ (current month sales) $+.60($ previous month sales $)$
Given this collection, the cash budget will be:

|  | April | May | June |
| :---: | :---: | :---: | :---: |
| Beginning cash balance | \$148,000 | \$194,400 | \$216,400 |
| Cash receipts |  |  |  |
| Cash collections from |  |  |  |
| credit sales | 305,600 | 314,800 | 320,400 |
| Total cash available | \$453,600 | \$509,200 | \$536,800 |
| Cash disbursements |  |  |  |
| Purchases | \$152,000 | \$168,000 | \$148,000 |
| Wages, taxes, and expenses | 75,200 | 68,800 | 84,000 |
| Interest | 8,000 | 8,000 | 8,000 |
| Equipment purchases | 24,000 | 48,000 | 152,000 |
| Total cash disbursements | \$259,200 | \$292,800 | \$392,000 |
| Ending cash balance | \$194,400 | \$216,400 | \$144,800 |

12. a. 45-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and $(90-45) / 90=1 / 2$ of current sales are collected.

| Beginning receivables | $\$ 3,300$ | $\$ 3,900$ | $\$ 4,250$ | $\$ 4,150$ |
| :--- | ---: | ---: | ---: | ---: |
| Sales | 7,800 | 8,500 | 8,300 | 9,700 |
| Cash collections | $\underline{-7,200}$ | $\underline{-8,150}$ | $\underline{-8,400}$ | $\underline{-9,000}$ |
| Ending receivables | $\underline{\$ 3,900}$ | $\underline{\$ 4,250}$ | $\underline{\underline{\$ 4,150}}$ | $\underline{\underline{\$ 4,850}}$ |

b. 60-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and $(90-60) / 90=1 / 3$ of current sales are collected.

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 3,300$ | $\$ 5,200$ | $\$ 5,667$ | $\$ 5,533$ |
| Sales | 7,800 | 8,500 | 8,300 | 9,700 |
| Cash collections | $\underline{-5,900}$ | $\underline{-8,033}$ | $\underline{-8,433}$ | $\underline{-8,767}$ |
| Ending receivables | $\underline{\$ 5,200}$ | $\underline{\underline{\$ 5,667}}$ | $\underline{\underline{\$ 5,533}}$ | $\underline{\underline{\$ 6,467}}$ |

c. 30-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and $(90-30) / 90=2 / 3$ of current sales are collected.

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 3,300$ | $\$ 2,600$ | $\$ 2,833$ | $\$ 2,767$ |
| Sales | 7,800 | 8,500 | 8,300 | 9,700 |
| Cash collections | $\underline{-8,500}$ | $\underline{-8,267}$ | $\underline{-8,367}$ | $\underline{-9,233}$ |
| Ending receivables | $\underline{\$ 2,600}$ | $\underline{\$ 2,833}$ | $\underline{\$ 2,767}$ | $\underline{\$ 3,233}$ |

## Intermediate

13. a. The EAR of the loan without the compensating balance is:

EAR $=(1+.0062)^{12}-1$
$\mathrm{EAR}=.07699$ or $7.699 \%$
However, with the compensating balance, you will only get the use of part of the amount, so the EAR with the compensating balance will be:
$\mathrm{EAR}=.07699 /(1-.03)$
$\mathrm{EAR}=.0794$ or $7.94 \%$
b. To end up with $\$ 15,000,000$, you must borrow:

Amount to borrow $=\$ 15,000,000 /(1-.03)$
Amount to borrow $=\$ 15,463,917.53$
The total interest you will pay on the loan is:
Total interest paid $=\$ 15,463,917.53(1.0062)^{6}-15,463,917.53$
Total interest paid $=\$ 584,248.28$

## B-276 SOLUTIONS

14. $a$. The EAR of your investment account is:

EAR $=1.008^{4}-1$
$\mathrm{EAR}=.0324$ or $3.24 \%$
b. To calculate the EAR of the loan, we can divide the interest on the loan by the amount of the loan. The interest on the loan includes the opportunity cost of the compensating balance. The opportunity cost is the amount of the compensating balance times the potential interest rate you could have earned. The compensating balance is only on the unused portion of the credit line, so:

Opportunity cost $=.04(\$ 75,000,000-40,000,000)(1.008)^{4}-.04(\$ 75,000,000-40,000,00)$ Opportunity cost $=\$ 45,340.47$

And the interest you will pay to the bank on the loan is:
Interest cost $=\$ 40,000,000(1.0158)^{4}-40,000,000$
Interest cost $=\$ 2,633,887.66$
So, the EAR of the loan in the amount of $\$ 40,000,000$ is:
EAR $=(\$ 45,340.47+2,633,887.66) / \$ 40,000,000$
$\mathrm{EAR}=.0658$ or $6.58 \%$
c. The compensating balance is only applied to the unused portion of the credit line, so the EAR of a loan on the full credit line is:
$\operatorname{EAR}=1.0158^{4}-1$
EAR $=.0647$ or $6.47 \%$
15. Here, we need to use the cash cycle and operating cycles to calculate the average accounts payable and average accounts receivable. We are given the cash cycle and the operating cycle, so the payables period is:

Cash cycle $=$ Operating cycle - Payables period
38 days $=53$ days - Accounts payable period
Accounts payable period $=15$ days
Now, we can use the payables period equation to find the accounts payable turnover, which is:
Payables period $=365$ / Payables turnover
$15=365 /$ Payables turnover
Payables turnover $=24.33$ times
Now, we can solve the payables turnover equation to find the average payables:
Payables turnover $=$ Cost of goods sold $/$ Average payables
$24.33=\$ 325,000 /$ Average payables
Average payables $=\$ 13,356.16$

Next, we can find the average accounts receivable. Using the equation for the operating cycle, the accounts payable period is:

Operating cycle $=$ Inventory period + Receivables period
53 days $=29$ days + Receivables period
Receivables period $=24$ days
Using the receivables period equation, we find:
Receivables period $=365$ / Receivables turnover
$24=365$ / Receivables turnover
Receivables turnover $=15.21$ times
Finally, the average accounts receivable balance is:
Receivables turnover $=$ Credit sales $/$ Average accounts receivable
$15.21=\$ 508,000 /$ Average accounts receivable
Average accounts receivable $=\$ 33,402.74$
16. Since the company has a 40 -day collection period, only those sales made in the first 50 days of the quarter will be collected in that quarter. Total cash collections in the first quarter will be:

Q1 cash collections $=$ Beginning receivables $+(50 / 90)($ Quarter 1 sales $)$
Q1 cash collections $=\$ 165+(50 / 90)(\$ 305)$
Q 1 cash collections $=\$ 334$
And cash collection in the second quarter will be sales made in the first 50 days of the quarter plus sales made in the last 40 days of the first quarter, so:

Q 2 cash collections $=(50 / 90)($ Quarter 2 sales $)+(40 / 90)($ Quarter 1 sales $)$
Q 2 cash collections $=(50 / 90)(\$ 420)+(40 / 90)(\$ 305)$
Q 2 cash collections $=\$ 369$
Quarter 3 and Quarter 4 collections will be:
Q3 cash collections $=(50 / 90)($ Quarter 3 sales $)+(40 / 90)($ Quarter 2 sales $)$
Q3 cash collections $=(50 / 90)(\$ 390)+(40 / 90)(\$ 420)$
Q3 cash collections $=\$ 403$
Q 4 cash collections $=(50 / 90)($ Quarter 4 sales $)+(40 / 90)($ Quarter 3 sales $)$
Q 4 cash collections $=(50 / 90)(\$ 330)+(40 / 90)(\$ 390)$
Q 4 cash collections $=\$ 421$
The beginning receivables in each quarter will be the sales made in the last 40 days of the previous quarter, so:

Q 2 beginning receivables $=(40 / 90)($ Quarter 1 sales $)$
Q2 beginning receivables $=(40 / 90)(\$ 305)$
Q 2 beginning receivables $=\$ 136$

## B-278 SOLUTIONS

Q3 beginning receivables $=(40 / 90)($ Quarter 2 sales $)$
Q3 beginning receivables $=(40 / 90)(\$ 420)$
Q3 beginning receivables $=\$ 187$
Q 4 beginning receivables $=(40 / 90)($ Quarter 3 sales $)$
Q4 beginning receivables $=(40 / 90)(\$ 390)$
Q 4 beginning receivables $=\$ 173$
The cash budget (in millions) for the company is:

|  | $Q 1$ | $Q 2$ | $Q 3$ | $Q 4$ |
| :--- | ---: | ---: | ---: | ---: |
| Beginning receivables | $\$ 165$ | $\$ 136$ | $\$ 187$ | $\$ 173$ |
| Sales | 305 | 420 | 390 | 445 |
| Cash collections | 334 | 369 | 403 | 421 |
| Ending receivables | $\$ 136$ | $\$ 187$ | $\$ 173$ | $\$ 198$ |
| Total cash collections | $\$ 334$ | $\$ 369$ | $\$ 403$ | $\$ 421$ |
| Total cash disbursements | 275 | 360 | 525 | 330 |
| Net cash inflow | $\$ 59$ | $\$ 9$ | $-\$ 122$ | $\$ 91$ |
| Beginning cash balance | $\$ 15$ | $\$ 74$ | $\$ 83$ | $-\$ 38$ |
| Net cash inflow | 59 | 9 | -122 | 91 |
| Ending cash balance | $\$ 74$ | $\$ 83$ | $-\$ 38$ | $\$ 52$ |
| Minimum cash balance | $-\$ 15$ | $-\$ 15$ | $-\$ 15$ | $-\$ 15$ |
| Cumulative surplus (deficit) | $\$ 59$ | $\$ 68$ | $-\$ 53$ | $\$ 37$ |

The company has a cash surplus for much of the year, but in Quarter 3, it will need to raise $\$ 53$ million to cover its cash flows.

## Challenge

17. $a$. For every dollar borrowed, you pay interest of:

Interest $=\$ 1(.016)=\$ 0.016$
You also must maintain a compensating balance of 4 percent of the funds borrowed, so for each dollar borrowed, you will only receive:

Amount received $=\$ 1(1-.04)=\$ 0.96$
We can adjust the EAR equation we have been using to account for the compensating balance by dividing the EAR by one minus the compensating balance, so:
$\mathrm{EAR}=\left[(1.016)^{4}-1\right] /(1-.04)$
$\mathrm{EAR}=.06828$ or $6.828 \%$

Another way to calculate the EAR is using the FVIF (or PVIF). For each dollar borrowed, we must repay:

Amount owed $=\$ 1(1.016)^{4}$
Amount owed $=\$ 1.0655$

At the end of the year the compensating will be returned, so your net cash flow at the end of the year will be:

End of year cash flow $=\$ 1.0655-.04$
End of year cash flow $=\$ 1.0255$

The present value of the end of year cash flow is the amount you receive at the beginning of the year, so the EAR is:

```
\(\mathrm{FV}=\mathrm{PV}(1+\mathrm{R})\)
\$1.0255 = \$0.96(1 + R)
\(\mathrm{R}=\$ 1.0255 / \$ 0.96-1\)
\(\mathrm{EAR}=.06828\) or \(6.828 \%\)
```

b. The EAR is the amount of interest paid on the loan divided by the amount received when the loan is originated. The amount of interest you will pay on the loan is the amount of the loan times the effective annual interest rate, so:

Interest $=\$ 210,000,000\left[(1.016)^{4}-1\right]$
Interest $=\$ 13,766,014.40$

For whatever loan amount you take, you will only receive 96 percent of that amount since you must maintain a 4 percent compensating balance on the portion of the credit line used. The credit line also has a fee of .125 percent, so you will only get to use:

Amount received $=.96(\$ 210,000,000)-.00125(\$ 400,000,000)$
Amount received $=\$ 201,100,000$

At the end of the loan, you will not repay the amount received since the commitment fee is a onetime fee paid up front. The amount you will repay (excluding interest) at the end of the loan will be the amount received plus the commitment, or:

Amount repaid $($ excluding interest $)=.96(\$ 210,000,000)$
Amount repaid (excluding interest) $=\$ 201,600,000$

So, the total amount repaid will be:
Total repaid $=\$ 201,600,000+13,766,014.40$
Total repaid $=\$ 215,366,014.40$

## B-280 SOLUTIONS

So, the EAR of the loan is:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{PV}(1+\mathrm{R}) \\
& \$ 215,366,014.40=\$ 201,100,000(1+\mathrm{R}) \\
& \mathrm{R}=.07094 \text { or } 7.094 \%
\end{aligned}
$$

18. You will pay interest of:

Interest $=\$ 15,000,000(.09)=\$ 1,350,000$
Additionally, the compensating balance on the loan is:
Compensating balance $=\$ 15,000,000(.05)=\$ 750,000$
Since this is a discount loan, you will receive the loan amount minus the interest payment. You will also not get to use the compensating balance. So, the amount of money you will actually receive on a $\$ 15$ million loan is:

Cash received $=\$ 15,000,000-1,350,000-750,000=\$ 12,900,000$
The EAR is the interest amount divided by the loan amount, so:

```
EAR = $1,350,000/$12,900,000
EAR = .1047 or 10.470%
```

We can also use the FVIF (or PVIF) here to calculate the EAR. Your cash flow at the beginning of the year is $\$ 12,990,000$. At the end of the year, your cash flow loan repayment, but you will also receive your compensating balance back, so:

End of year cash flow $=\$ 15,000,000-750,000$
End of year cash flow $=\$ 14,250,000$
So, using the time value of money, the EAR is:

```
$14,250,000 = $12,900,000(1 + R)
R = $14,250,000/$12,900,000 - 1
EAR = .1047 or 10.47%
```


## CHAPTER 17 WORKING CAPITAL MANAGEMENT

## Answers to Concepts Review and Critical Thinking Questions

1. Yes. Once a firm has more cash than it needs for operations and planned expenditures, the excess cash has an opportunity cost. It could be invested (by shareholders) in potentially more profitable ways. Question 9 discusses another reason.
2. If it has too much cash, it can simply pay a dividend, or, more likely in the current financial environment, buy back stock. It can also reduce debt. If it has insufficient cash, then it must either borrow, sell stock, or improve profitability.
3. Probably not. Creditors would probably want substantially more.
4. Auto manufacturers often argue that due to the cyclical nature of their business, cash reserves are a good way to deal with future economic downturns. This is debatable, but it is true that auto manufacturers' operating cash flows are very sensitive to the business cycle, and enormous losses have occurred during recent downturns.
5. Such instruments go by a variety of names, but the key feature is that the dividend adjusts, keeping the price relatively stable. This price stability, along with the dividend tax exemption, makes so-called adjustable rate preferred stock very attractive relative to interest-bearing instruments.
6. Net disbursement float is more desirable because the bank thinks the firm has more money than it actually does, and the firm is therefore receiving interest on funds it has already spent.
7. The firm has a net disbursement float of $\$ 500,000$. If this is an ongoing situation, the firm may be tempted to write checks for more than it actually has in its account.
8. $a$. About the only disadvantage to holding T-bills are the generally lower yields compared to alternative money market investments.
b. Some ordinary preferred stock issues pose both credit and price risks that are not consistent with most short-term cash management plans.
c. The primary disadvantage of NCDs is the normally large transactions sizes, which may not be feasible for the short-term investment plans of many smaller to medium-sized corporations.
d. The primary disadvantages of the commercial paper market are the higher default risk characteristics of the security, and the lack of an active secondary market which may excessively restrict the flexibility of corporations to meet their liquidity adjustment needs.
9. The concern is that excess cash on hand can lead to poorly thought-out investments. The thought is that keeping cash levels relatively low forces management to pay careful attention to cash flow and capital spending.
10. A potential advantage is that the quicker payment often means a better price. The disadvantage is that doing so increases the firm's cash cycle.

## B-282 SOLUTIONS

11. This is really a capital structure decision. If the firm has an optimal capital structure, paying off debt moves it to an under-leveraged position. However, a combination of debt reduction and stock buybacks could be structured to leave capital structure unchanged.
12. It is unethical because you have essentially tricked the grocery store into making you an interest-free loan, and the grocery store is harmed because it could have earned interest on the money instead of loaning it to you.
13. $a$. A sight draft is a commercial draft that is payable immediately.
b. A time draft is a commercial draft that does not require immediate payment.
c. A banker's acceptance is when a bank guarantees the future payment of a commercial draft.
d. A promissory note is an IOU that the customer signs.
$e$. A trade acceptance is when the buyer accepts the commercial draft and promises to pay it in the future.
14. Trade credit is usually granted on open account. The invoice is the credit instrument.
15. The costs of granting credit are carrying costs, namely, the cost of debt, possibility of default, and the cash discount. The major cost of not grating credit is the opportunity cost of lost sales. The sum of these costs for different levels of receivables creates the total credit cost curve.
16. 17. Character: determines if a customer is willing to pay his or her debts.
1. Capacity: determines if a customer is able to pay debts out of operating cash flow.
2. Capital: determines the customer's financial reserves in case problems occur with operating cash flow.
3. Collateral: assets that can be liquidated to pay off the loan in case of default.
4. Conditions: customer's ability to weather an economic downturn and whether such a downturn is likely.
5. 6. Perishability and collateral value
1. Consumer demand
2. Cost, profitability, and standardization
3. Credit risk
4. The size of the account
5. Competition
6. Customer type

If the credit period exceeds a customer's operating cycle, then the firm is financing the receivables and other aspects of the customer's business that go beyond the purchase of the selling firm's merchandise.
18. a. B: A is likely to sell for cash only, unless the product really works. If it does, then they might grant longer credit periods to entice buyers.
b. A: Landlords have significantly greater collateral, and that collateral is not mobile.
c. A: Since A's customers turn over inventory less frequently, they have a longer inventory period and thus will most likely have a longer credit period as well.
d. B: Since A's merchandise is perishable and B's is not, B will probably have a longer credit period.
e. A: Rugs are fairly standardized and they are transportable, while carpets are custom fit and are not particularly transportable.
19. The three main categories of inventory are: raw material (initial inputs to the firm's production process), work-in-progress (partially completed products), and finished goods (products ready for sale). From the firm's perspective, the demand for finished goods is independent of the demand for the other types of inventory. The demand for raw material and work-in-progress is derived from, or dependent on, the firm's needs for these inventory types in order to achieve the desired levels of finished goods.
20. JIT systems reduce inventory amounts. Assuming no adverse effects on sales, inventory turnover will increase. Since assets will decrease, total asset turnover will also increase. Recalling the Du Pont equation, an increase in total asset turnover, all else being equal, has a positive effect on ROE.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. The available balance is the amount you have on deposit, or $\$ 165,000$. By writing a check, you now have a disbursement float. Your available balance is the amount on deposit minus the amount of the check, or:

Amount available $=\$ 165,000-48,000$
Amount available $=\$ 117,000$
2. The available balance is the amount you have on deposit, or $\$ 11,500$. This is a collection float since you are waiting for the deposited check to clear into your account. The book balance is the amount on deposit plus the collection float, or:

Book balance $=\$ 11,500+6,200$
Book balance $=\$ 17,700$

## B-284 SOLUTIONS

3. Your disbursement float is the amount of the check you wrote, or $\$ 5,100$. The collection float is the amount of the check deposited, or $-\$ 8,400$. The net float is the sum of the disbursement float and collection float, or:

Net float $=\$ 5,100-8,400$
Net float $=-\$ 3,300$
4. a. There are 40 days until account is overdue. If you take the full period, you must remit:

$$
\begin{aligned}
& \text { Remittance }=900(\$ 58) \\
& \text { Remittance }=\$ 52,200
\end{aligned}
$$

b. There is a 1 percent discount offered, with a 25 day discount period. If you take the discount, you will only have to remit:

```
Remittance = (1-.01)($52,200)
Remittance = $51,678
```

c. The implicit interest is the difference between the two remittance amounts, or:

Implicit interest $=\$ 52,200-51,678$
Implicit interest $=\$ 522$
The number of days' credit offered is:
Days' credit $=40-25$
Days' credit $=15$ days
5. The average daily receipts are the total amount of checks received divided by the number of days in a month. Assuming 30 days in a month, the average daily float is:

Average daily receipts $=\$ 67,000 / 30$
Average daily receipts $=\$ 2,233.33$
This is the average amount of checks received per day. The average check takes three days to clear, so the average daily float is:

Average daily float $=3(\$ 2,233.33)$
Average daily float $=\$ 6,700$
6. a. The disbursement float is the average monthly checks written times the average number of days for the checks to clear, so:

Disbursement float $=4(\$ 46,000)$
Disbursement float $=\$ 184,000$

The collection float is the average monthly checks received times the average number of days for the checks to clear, so:

Collection float $=2(-\$ 41,000)$
Collection float $=-\$ 142,000$
The net float is the disbursement float plus the collection float, so:
Net float $=\$ 184,000-142,000$
Net float $=\$ 42,000$
b. The new collection float will be:

Collection float $=1(-\$ 71,000)$
Collection float $=-\$ 71,000$
And the new net float will be:

$$
\begin{aligned}
& \text { Net float }=\$ 156,000-71,000 \\
& \text { Net float }=\$ 113,000
\end{aligned}
$$

7. The total sales of the firm are equal to the total credit sales since all sales are on credit, so:

Total credit sales $=4,000(\$ 400)$
Total credit sales $=\$ 1,600,000$
The average collection period is the percentage of accounts taking the discount times the discount period, plus the percentage of accounts not taking the discount times the days' until full payment is required, so:

Average collection period $=.40(10)+.60(30)$
Average collection period $=22$ days
The receivables turnover is 365 divided by the average collection period, so:
Receivables turnover $=365 / 22$
Receivables turnover $=16.591$ times
And the average receivables are the credit sales divided by the receivables turnover so:
Average receivables $=\$ 1,600,000 / 16.591$
Average receivables $=\$ 96,438.36$
If the firm increases the cash discount, more people will pay sooner, thus lowering the average collection period. If the ACP declines, the receivables turnover increases, which will lead to a decrease in the average receivables.

## B-286 SOLUTIONS

8. The receivables turnover is 365 divided by the average collection period, so:

Receivables turnover $=365 / 34$
Receivables turnover $=10.735$ times

And the average receivables are the credit sales divided by the receivables turnover so:
Average receivables $=\$ 57,000,000 / 10.735$
Average receivables $=\$ 5,309,589.04$
9. $a$. The average collection period is the percentage of accounts taking the discount times the discount period, plus the percentage of accounts not taking the discount times the days' until full payment is required, so:

Average collection period $=.60(10$ days $)+.40(30$ days $)$
Average collection period $=18$ days
b. And the average daily balance is:

Average balance $=1,200(\$ 2,300)(12)(18 / 365)$
Average balance $=\$ 1,633,315.07$
10. The daily sales are:

Daily sales $=\$ 44,500 / 7$
Daily sales $=\$ 6,357.14$

Since the average collection period is 25 days, the average accounts receivable is:
Average accounts receivable $=\$ 6,357.14(25)$
Average accounts receivable $=\$ 158,928.57$
11. The interest rate for the term of the discount is:

Interest rate $=.02 / .98$
Interest rate $=.0204$ or $2.04 \%$

And the interest is for:
$40-10=30$ days
So, using the EAR equation, the effective annual interest rate is:

EAR $=(1+\text { Periodic rate })^{\mathrm{m}}-1$
EAR $=(1.0204)^{365 / 30}-1$
$\mathrm{EAR}=.2786$ or $27.86 \%$
a. The periodic interest rate is:

Interest rate $=.03 / .97$
Interest rate $=.0309$ or $3.09 \%$
And the EAR is:
$\operatorname{EAR}=(1.0309)^{365 / 30}-1$
EAR $=.4486$ or $44.86 \%$
b. The EAR is:
$\operatorname{EAR}=(1.0204)^{365 / 50}-1$
$\mathrm{EAR}=.1589$ or $=15.89 \%$
c. The EAR is:
$\operatorname{EAR}=(1.0204)^{365 / 20}-1$
$\operatorname{EAR}=.4459$ or $44.59 \%$
12. The receivables turnover is:

Receivables turnover $=365 /$ Average collection period
Receivables turnover $=365 / 37$
Receivables turnover $=9.865$ times
And the annual credit sales are:
Annual credit sales $=$ Receivables turnover $\times$ Average daily receivables
Annual credit sales $=9.865(\$ 91,000)$
Annual credit sales $=\$ 897,702.70$
13. The carrying costs are the average inventory times the cost of carrying an individual unit, so:

Carrying costs $=(1,600 / 2)(\$ 4)$
Carrying costs $=\$ 3,200$
The order costs are the number of orders times the cost of an order, so:
Restocking costs $=52(\$ 650)$
Restocking costs $=\$ 33,800$
The economic order quantity is:

```
EOQ = [(2T }\times\textrm{F})/\textrm{CC}\mp@subsup{]}{}{1/2
EOQ = [2(52)(1,600)($650)/$4] ]
EOQ = 5,200
```


## B-288 SOLUTIONS

The number of orders per year will be the total units sold per year divided by the EOQ, so:
Number of orders per year $=52(1,600) / 5,200$
Number of orders per year $=16$
The firm's policy is not optimal, since the carrying costs and the order costs are not equal. The company should increase the order size and decrease the number of orders.
14. The carrying costs are the average inventory times the cost of carrying an individual unit, so:

Carrying costs $=(900 / 2)(\$ 38)$
Carrying costs $=\$ 17,100$
The order costs are the number of orders times the cost of an order, so:
Restocking costs $=12(\$ 530)$
Restocking costs $=\$ 6,360$
The economic order quantity is:

```
\(\mathrm{EOQ}=[(2 \mathrm{~T} \times \mathrm{F}) / \mathrm{CC}]^{1 / 2}\)
\(\mathrm{EOQ}=[2(12)(900)(\$ 530) / \$ 38]^{1 / 2}\)
\(\mathrm{EOQ}=548.87\)
```

The number of orders per year will be the total units sold per year divided by the EOQ, so:
Number of orders per year $=12(900) / 548.87$
Number of orders per year $=19.68$

The firm's policy is not optimal, since the carrying costs and the order costs are not equal. The company should decrease the order size and increase the number of orders.

## Intermediate

15. The total carrying costs are:

Carrying costs $=(\mathrm{Q} / 2) \times \mathrm{CC}$
where CC is the carrying cost per unit. The restocking costs are:
Restocking costs $=\mathrm{F} \times(\mathrm{T} / \mathrm{Q})$
So, the total cost is:
Total cost $=$ Carrying cost + Restocking costs
Total cost $=(\mathrm{Q} / 2) \times \mathrm{CC}+\mathrm{F} \times(\mathrm{T} / \mathrm{Q})$

Using calculus to find the minimum point of the curve, we take the derivative and set it equal to zero. Doing so, we find:

$$
\begin{aligned}
& \partial / \partial \mathrm{Q}=0=(\mathrm{CC} / 2)+\left(\mathrm{F} \times \mathrm{T} \times-\mathrm{Q}^{-2}\right) \\
& -\mathrm{Q}^{-2}=-\mathrm{CC} /(2 \times \mathrm{F} \times \mathrm{T}) \\
& \mathrm{Q}^{-2}=\mathrm{CC} /(2 \times \mathrm{F} \times \mathrm{T}) \\
& \mathrm{Q}^{2}=(2 \times \mathrm{F} \times \mathrm{T}) / \mathrm{CC} \\
& \mathrm{Q}=[(2 \times \mathrm{F} \times \mathrm{T}) / \mathrm{CC}]^{1 / 2}
\end{aligned}
$$

To prove this point is a minimum, we can find the second derivative, which is:
$\left.\partial / \partial \mathrm{Q}\left[(\mathrm{CC} / 2)+\mathrm{F} \times \mathrm{T} \times-\mathrm{Q}^{-2}\right)\right]=\mathrm{F} \times \mathrm{T} \times 2 \mathrm{Q}^{-3}=(2 \times \mathrm{F} \times \mathrm{T}) / \mathrm{Q}^{3}$
Since the second derivative is greater than zero so long as $\mathrm{F}, \mathrm{T}$, and Q are all positive, the first derivative is at a minimum.

## Challenge

16. The company places an order every five days. The number of orders per year will be:

Orders per year $=365 / 5=73$ times
The next order should be placed after the close of business Saturday.

## CHAPTER 18 INTERNATIONAL ASPECTS OF FINANCIAL MANAGEMENT

## Answers to Concepts Review and Critical Thinking Questions

1. a. The dollar is selling at a premium, because it is more expensive in the forward market than in the spot market (SF 1.53 versus SF 1.50).
b. The franc is expected to depreciate relative to the dollar, because it will take more francs to buy one dollar in the future than it does today.
c. Inflation in Switzerland is higher than in the United States, as are interest rates.
2. The exchange rate will increase, as it will take progressively more rubles to purchase a dollar as the higher inflation in Russia will devalue the ruble. This is the relative PPP relationship.
3. $a$. The Australian dollar is expected to weaken relative to the dollar, because it will take more $\mathrm{A} \$$ in the future to buy one dollar than it does today.
b. The inflation rate in Australia should be higher.
c. Nominal interest rates in Australia should be higher; relative real rates in the two countries should be the same.
4. A Yankee bond is most accurately described by d.
5. Either. For example, if a country's currency strengthens, imports become cheaper (good), but its exports become more expensive for others to buy (bad). The reverse is true for a currency depreciation.
6. The main advantage is the avoidance of the tariff. There are probably other advantages, but we don't know about them. Disadvantages include political risk, the different exchange rate risk as Hynix now has to be concerned with the Taiwanese exchange rate as well, and costs of supervising distant operations, although ProMOS will provide much of the supervision.
7. One key thing to remember is that dividend payments are made in the home currency. More generally, it may be that the owners of the multinational are primarily domestic who are ultimately concerned about their wealth denominated in their home currency because, unlike a multinational, they are not internationally diversified.
8. a. False. If prices are rising faster in Great Britain, it will take more pounds to buy the same amount of goods that one dollar can buy; the pound will depreciate relative to the dollar.
b. False. The forward market would already reflect the projected deterioration of the euro relative to the dollar. Only if you feel that there might be additional, unanticipated weakening of the euro that isn't reflected in forward rates today will the forward hedge protect you against additional declines.
c. True. The market would only be correct on average, while you would be correct all the time.
9. a. American exporters: their situation in general improves because a sale of the exported goods for a fixed number of pesos will be worth more dollars.
American importers: their situation in general worsens because the purchase of the imported goods for a fixed number of pesos will cost more in dollars.
b. American exporters: they would generally be better off if the British government's intentions result in a strengthened pound.
American importers: they would generally be worse off if the pound strengthens.
c. American exporters: would generally be much worse off, because an extreme case of fiscal expansion like this one will make American goods prohibitively expensive to buy, or else Brazilian sales, if fixed in reais, would become worth an unacceptably low number of dollars. American importers: would generally be much better off, because Brazilian goods will become much cheaper to purchase in dollars.
10. False. If the financial markets are perfectly competitive, the difference between the Eurodollar rate and the U.S. rate will be due to differences in risk and government regulation. Therefore, speculating in those markets will not be beneficial.

## Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

## Basic

1. Using the quotes from the table, we get:
a. $\quad \$ 100(\mathrm{Z} 2.9949 / \$ 1)=\mathrm{Z} 299.49$
b. $\$ 1.3020$

## B-292 SOLUTIONS

c. $€ 5,000,000(\$ 1.3020 / €)=\$ 6,510,000$

Alternatively the question can be answered as:
$€ 5,000,000 /(\$ .7680 / €)=\$ 6,510,417$
The difference is due to rounding in the exchange rate quote.
d. New Zealand
e. Mexican peso
f. $\quad(\mathrm{SF} 1.2439 / \$)(\$ 1.3020 / €)=1.6196 \mathrm{SF} / €$
g. Most valuable: Kuwaiti dinar $=\$ 3.4577$

Least valuable: Indonesian rupiah $=\$ 0.0001101$
2. a. You would prefer $£ 100$, since:
$(£ 100)(\$ 1.9669 / £ 1)=\$ 196.69$
b. You would still prefer $£ 100$. Using the $\$ / £$ exchange rate and the $\mathrm{C} \$ / \$$ exchange rate to find the amount of Canadian dollars $£ 100$ will buy, we get:
$(£ 100)(\$ 1.9669 / £ 1)(\mathrm{C} \$ 1.1788 / \$ 1)=\mathrm{C} \$ 231.86$
c. Using the quotes in the book to find the $\mathrm{C} \$ / £$ cross rate, we find:
$(\mathrm{C} \$ 1.1788 / \$ 1)(\$ 1.9669 / £ 1)=\mathrm{C} \$ 2.3186 / £ 1$
The $£ / \mathrm{C} \$$ exchange rate is the inverse of the $\mathrm{C} \$ / £$ exchange rate, so:
$£ 1 / \mathrm{C} \$ 2.3186=£ 0.4313 / \mathrm{C} \$$
3. a. $\quad \mathrm{F}_{180}=¥ 117.87$ (per $\$$ ). The yen is selling at a premium because it is more expensive in the forward market than in the spot market ( $\$ 0.0082843$ versus $\$ 0.0084839$ ).
b. $\quad \mathrm{F}_{90}=\$ 0.8505 / \mathrm{C} \$$. The U.S. dollar is selling at a discount because it is less expensive in the forward market than in the spot market ( $\mathbf{C} \$ 1.17883$ versus $\mathrm{C} \$ 1.17578$ ).
c. The value of the dollar will fall relative to the yen, since it takes more dollars to buy one yen in the future than it does today. The value of the dollar will fall relative to the Canadian dollar, because it will take more US dollars to buy the Canadian dollar in the future than it does today.
4. $a$. The U.S. dollar, since one Canadian dollar will buy:
$(\operatorname{Can} \$ 1) /(\operatorname{Can} \$ 1.19 / \$ 1)=\$ 0.8403$
b. The cost in U.S. dollars is:
$(\operatorname{Can} \$ 3.10) /(\operatorname{Can} \$ 1.19 / \$ 1)=\$ 2.61$
Among the reasons that absolute PPP doesn't hold are tariffs and other barriers to trade, transactions costs, taxes, and different tastes.
c. The U.S. dollar is selling at a premium because it is more expensive in the forward market than in the spot market (Can\$1.19 versus Can\$1.24).
d. The Canadian dollar is expected to depreciate in value relative to the dollar, because it takes more Canadian dollars to buy one U.S. dollar in the future than it does today.
e. Interest rates in the United States are probably lower than they are in Canada.
5. a. The cross rate in $¥ / £$ terms is:
$(¥ 123 / \$ 1)(\$ 1.92 / £ 1)=¥ 236.16 / £ 1$
b. The yen is quoted too low relative to the pound. Take out a loan for $\$ 1$ and buy $¥ 123$. Use the $¥ 123$ to purchase pounds at the cross-rate, which will give you:
$¥ 123(£ 1 / \not \approx 215)=£ 0.57209$
Use the pounds to buy back dollars and repay the loan. The cost to repay the loan will be:
$£ 0.57209(\$ 1.92 / £ 1)=\$ 1.0984$
Your arbitrage profit is $\$ 0.0984$ per dollar used.
6. We can rearrange the approximate interest rate parity condition to answer this question. The equation we will use is:
$\mathrm{R}_{\mathrm{FC}}=\left(\mathrm{F}_{\mathrm{t}}-\mathrm{S}_{0}\right) / \mathrm{S}_{0}+\mathrm{R}_{\mathrm{US}}$
Using this relationship, we find:
Canada: $\quad \mathrm{R}_{\mathrm{FC}}=(\mathrm{C} \$ 1.1727-\mathrm{C} \$ 1.1788) / \mathrm{C} \$ 1.1788+.05=.0448$ or $4.48 \%$
Japan: $\quad \mathrm{R}_{\mathrm{FC}}=(¥ 117.87-¥ 120.71) \not \approx 120.71+.05=.0265$ or $2.65 \%$
Great Britain: $\quad \mathrm{R}_{\mathrm{FC}}=(£ 0.5091-£ 0.5084) / £ 0.5084+.05=.0514$ or $5.14 \%$

## B-294 SOLUTIONS

7. If we invest in the U.S. for the next three months, we will have:
$\$ 30,000,000(1.0047)^{3}=\$ 30,424,991.21$
If we invest in Great Britain, we must exchange the dollars today for pounds, and exchange the pounds for dollars in three months. After making these transactions, the dollar amount we would have in three months would be:
$(\$ 30,000,000)(£ 0.52 / \$ 1)(1.0019)^{3} /(£ 0.51 / \$ 1)=\$ 30,762,919.72$
We should invest in Great Britain.
8. Using the relative purchasing power parity equation:
$\mathrm{F}_{\mathrm{t}}=\mathrm{S}_{0} \times\left[1+\left(\mathrm{h}_{\mathrm{FC}}-\mathrm{h}_{\mathrm{US}}\right)\right]^{\mathrm{t}}$
We find:

$$
\begin{aligned}
& \mathrm{R} 29.47=\mathrm{R} 27.05\left[1+\left(\mathrm{h}_{\mathrm{FC}}-\mathrm{h}_{\mathrm{US}}\right)\right]^{3} \\
& \mathrm{~h}_{\mathrm{FC}}-\mathrm{h}_{\mathrm{US}}=(\mathrm{R} 29.47 / \mathrm{R} 27.05)^{1 / 3}-1 \\
& \mathrm{~h}_{\mathrm{FC}}-\mathrm{h}_{\mathrm{US}}=.0290 \text { or } 2.90 \%
\end{aligned}
$$

Inflation in Russia is expected to exceed that in the U.S. by $2.90 \%$ over this period.
9. The profit will be the quantity sold, times the sales price minus the cost of production. The production cost is in Singapore dollars, so we must convert this to U.S. dollars. Doing so, we find that if the exchange rates stay the same, the profit will be:

$$
\begin{aligned}
& \text { Profit }=30,000[\$ 160-\{(\mathrm{S} \$ 229.50) /(\mathrm{S} \$ 1.5342 / \$ 1)\}] \\
& \text { Profit }=\$ 312,319.12
\end{aligned}
$$

If the exchange rate rises, we must adjust the cost by the increased exchange rate, so:

$$
\begin{aligned}
& \text { Profit }=30,000[\$ 160-\{(\mathrm{S} \$ 229.50) / 1.1(\mathrm{~S} \$ 1.5342 / \$ 1)\}] \\
& \text { Profit }=\$ 720,290.11
\end{aligned}
$$

If the exchange rate falls, we must adjust the cost by the decreased exchange rate, so:

```
Profit = 30,000[$160 - {(S$229.50)/0.9(S$1.5342/$1)}]
Profit = $186,312.08
```

To calculate the breakeven change in the exchange rate, we need to find the exchange rate that make the cost in Singapore dollars equal to the selling price in U.S. dollars, so:

```
$160 = S$229.50/S
ST}=\textrm{S}$1.4344/$
```

This is a decline of:
Maximum decline $=(\mathrm{S} \$ 1.4344-1.5342) / \mathrm{S} \$ 1.5342$
Maximum decline $=-.0651$ or $-6.51 \%$
10. a. If IRP holds, then:
$\mathrm{F}_{180}=(\mathrm{W} 950.18)[1+(.07-.05)]^{1 / 2}$
$\mathrm{F}_{180}=\mathrm{W} 959.6348$
Since given $\mathrm{F}_{180}$ is W956.85, an arbitrage opportunity exists; the forward premium is too low. Borrow \$1 today at 5\% interest. Agree to a 180 -day forward contract at W956.85. Convert the loan proceeds into won:

W1 (\$1/W950.18) = W950.18
Invest these won at 7\%, ending up with W982.419. Convert the won back into dollars as
W982.419(\$1/W956.85) = \$1.0267
Repay the $\$ 1$ loan, ending with a profit of:

$$
\$ 1.0267-1.02435=\$ 0.00237
$$

b. To find the forward rate that eliminates arbitrage, we use the interest rate parity condition, so:

$$
\begin{aligned}
& \mathrm{F}_{180}=(\mathrm{W} 950.18)[1+(.07-.05)]^{1 / 2} \\
& \mathrm{~F}_{180}=\mathrm{W} 959.6348
\end{aligned}
$$

11. $a$. The yen is expected to get stronger, since it will take fewer yen to buy one dollar in the future than it does today.
b. $\mathrm{h}_{\mathrm{US}}-\mathrm{h}_{\mathrm{JAP}} \approx(¥ 116.83-¥ 119.51) \not \approx \neq 119.51$
$\mathrm{h}_{\mathrm{US}}-\mathrm{h}_{\mathrm{JAP}}=-.0224$ or $-2.24 \%$
$(1-.0224)^{4}-1=-.0867$ or $-8.67 \%$
The approximate inflation differential between the U.S. and Japan is $-8.67 \%$ annually.
12. We need to find the change in the exchange rate over time, so we need to use the interest rate parity relationship:
$\mathrm{F}_{\mathrm{t}}=\mathrm{S}_{0} \times\left[1+\left(\mathrm{R}_{\mathrm{FC}}-\mathrm{R}_{\mathrm{US}}\right)\right]^{\mathrm{t}}$
Using this relationship, we find the exchange rate in one year should be:
$\mathrm{F}_{1}=197[1+(.087-.048)]^{1}$
$\mathrm{F}_{1}=$ HUF 204.68

## B-296 SOLUTIONS

The exchange rate in two years should be:
$\mathrm{F}_{2}=197[1+(.087-.048)]^{2}$
$\mathrm{F}_{2}=$ HUF 221.67
And the exchange rate in five years should be:
$\mathrm{F}_{5}=197[1+(.087-.048)]^{5}$
$\mathrm{F}_{5}=$ HUF 238.53
13. Pounds are cheaper in New York, so we start there. Buy:
$\$ 10,000(£ / \$ 1.9684)=£ 5,080.268$
in New York. Sell the $£ 5,080.268$ in London for
$£ 5,080.268(\$ 1.9735 / £)=\$ 10,025.91$
Your profit is $\$ 10,025.91-10,000=\$ 25.91$
for each $\$ 10,000$ transaction.
14. If purchasing power parity holds, the exchange rate will be:

Kronur229/\$3.29 = kronur69.6049/\$
15. a. To construct the balance sheet in dollars, we need to convert the account balances to dollars. At the current exchange rate, we get:

Assets $=$ solaris $15,000 \times(\$ /$ solaris 1.20$)=\$ 12,500$
Debt $=$ solaris $6,000 \times(\$ /$ solaris 1.20$)=\$ 5,000$
Equity $=$ solaris $9,000 \times(\$ /$ solaris 1.20$)=\$ 7,500$
b. In one year, if the exchange rate is solaris $1.40 / \$$, the accounts will be:

Assets $=$ solaris $15,000 \times(\$ /$ solaris 1.40$)=\$ 10,714.29$
Debt $=$ solaris $6,000 \times(\$ /$ solaris 1.40$)=\$ 4,285.71$
Equity $=$ solaris $9,000 \times(\$ /$ solaris 1.40$)=\$ 6,428.57$
$b$. If the exchange rate is solaris $1.15 / \$$, the accounts will be:
Assets $=$ solaris $15,000 \times(\$ /$ solaris 1.15$)=\$ 13,043.48$
Debt $=$ solaris $6,000 \times(\$ /$ solaris 1.15$)=\$ 5,217.39$
Equity $=$ solaris $9,000 \times(\$ /$ solaris 1.15$)=\$ 7,826.09$

## Challenge

16. First, we need to construct the end of year balance sheet in solaris. Since the company has retained earnings, the equity account will increase, which necessarily implies the assets will also increase by the same amount. So, the balance sheet at the end of the year in solaris will be:

|  | Balance Sheet (solaris) |  |
| :--- | :--- | ---: |
|  | Liabilities | $\$ 6,000.00$ |
|  | Equity | $\underline{10,100.00}$ |
| Assets | $\underline{\$ 16,100.00}$ | Total liabilities \& equity |

Now we need to convert the balance sheet accounts to dollars, which gives us:
Assets $=$ solaris $16,100 \times(\$ /$ solaris 1.24$)=\$ 12,983.87$
Debt $=$ solaris $6,000 \times(\$ /$ solaris 1.24$)=\$ 4,838.71$
Equity $=$ solaris $10,100 \times(\$ /$ solaris 1.24$)=\$ 8,145.16$

